

2024 SUBJECT WORKBOOK

Grade 12

$a+b=c$ MATHEMATICAL LITERACY

A joint initiative between the Western Cape Education Department and Stellenbosch University.

BROADCAST SESSIONS

GRADE 12

TARIFF SYSTEMS

GRADE 12

MAPS AND PLANS

Session	Date	Time	Topic
1	13/02/2024	15h00-16h00	Tariff Systems
2	01/08/2024	16h00-17h00	Maps and Plans: Scale

INTRODUCTION AND TOPICS

TARIFF SYSTEMS

A tariff is the charge in rands per measuring unit for a specific service, such as electricity, water, transport, or telephone.

Both households and businesses commonly have to pay for services according to certain tariffs or charges.

$$\text{Tariff (cost per unit)} = \frac{\text{total cost}}{\text{number of units}}$$

Maps and Plans: Scale

Scale drawings represent the actual size and shape of an object on paper.

Topics	Description
TARIFF SYSTEMS	Tariffs are not always constant, for example: the price of electricity and water per unit gets more expensive the more electricity and water you use. Water consumption is measured in kilolitres (kℓ) and electricity consumption in kilowatt-hours (kWh).
Maps and Plans: Scale	Each dimension of the actual object is either reduced (for very large objects e.g. a building) or enlarged (for very small objects e.g. a screw) by a certain ratio, called the scale factor.

TERMINOLOGY

Term	Definition
Account	A record of income and expenditure.
Tariff	The rate charged for a service rendered
Taxable	A service, purchase or item or earning that has tax applied to it.
VAT	Value Added Tax (VAT) is a tax that is levied at 15% (currently in South Africa) on most goods and services, as well as on the importation of goods and services into South Africa.
VAT exclusive price	The price before VAT is added.
VAT inclusive price	The price after VAT is added.
Bar scales	Presented as a picture, it means that if you placed a ruler next to this scale, you could determine how many centimeters next to this scale, you could determine how many centimeters represent the specified kilometers
Number scale	A number scale such as 1 : 50 000 means that 1 unit on the map represent 50 000 units in real life.
Scale	Determines how many times smaller an object shown on a plan or map is that its actual size.

SESSION 1 | TARIFF SYSTEMS



WHAT YOU SHOULD KNOW

How to determine water and electricity tariffs.

First calculate the kℓ per interval:

We have used 38 kℓ of water at home.

Subtract the endpoints of the interval to get the maximum amount of kℓ used per bracket:

Interval 1: 6kℓ used

Interval 2: $10 - 6 = 4$

Interval 3: $20 - 10 = 10$

Interval 4: $35 - 20 = 15$

Interval 5: $50 - 35 = 15$ The amount use in this interval is the amount above 35

etc. $38 - 35 = 3$

Adding the kilolitres used = $6+4+10+15+3 = 38$ kℓ

Remember:

1. Use the given table.
2. Calculate the water per step/interval
3. Check if the volume/amount of water adds up to the volume/amount of water used.
4. Multiply with the tariff per step/interval
5. Add the amounts in Rand per interval to get the total cost.

EXAMPLE 1

The TABLE below shows the stepped water tariff rates (sliding scale) for residential properties in Cape Town. As from 1 February 2018 level 6 tariffs were charged.

TABLE: Stepped water tariff rates (sliding scale) for residential households in Cape Town (Adapted)

STEP	VOLUME/AMOUNT OF WATER USED (1 kℓ = 1 000 LITRES)	LEVEL 4 R/kℓ (INCLUDING VAT – 15%)	LEVEL 6 R/kℓ (INCLUDING VAT – 15%)
1	more than 0 kℓ to 6 kℓ	R4,65	R29,93
2	above 6 kℓ to 10 kℓ	R17,75	R52,44
3	above 10 kℓ up to 20 kℓ	R25,97	R114,00
4	above 20 kℓ up to 35 kℓ	R43,69	R134,00
5	above 35 kℓ up to 50 kℓ	R113,99	R912,00
6	more than 50 kℓ	R302,24	R912,00

Use the Table above to answer the questions that follow:

1. What is the tariff on LEVEL 4 if a household used an amount of water that is above 20 kℓ up to 35 kℓ (Step 4)?
2. In which STEP/INTERVAL will you pay R52,44 on Level 6?
3. Calculate the VAT exclusive tariff of STEP 1 of Level 6.
4. Use the table to calculate the amount a household will have to pay on Level 4 for consuming 15 kℓ.
5. Use the table to calculate the amount a household will have to pay on Level 6 for consuming 23 kℓ.

ANSWERS

1. R43,69
2. Step 2 or above 6 kℓ to 10,5 kℓ
3. VAT Exclusive tariff of STEP 1 LEVEL 6 = $R29,93 \div 1,15 \approx R26,03$

TARIFF SYSTEMS

4.

STEP	VOLUME/AMOUNT OF WATER USED (1 kℓ = 1 000 LITRES)	LEVEL 4 R/kℓ (INCLUDING VAT –15%)
1	more than 0 kℓ to 6 kℓ	$6 \times R4,65 = 27,90$
2	above 6 kℓ to 10 kℓ	$4 \times R17,75 = 71,00$
3	above 10 kℓ up to 20 kℓ	$5 \times R25,97 = 129,85$
4	above 20 kℓ up to 35 kℓ	
5	above 35 kℓ up to 50 kℓ	
6	more than 50 kℓ	
Total: 15		R228,75

5.

STEP	VOLUME/AMOUNT OF WATER USED (1 kℓ = 1 000 LITRES)	LEVEL 6 R/kℓ (INCLUDING VAT–15%)
1	more than 0 kℓ to 6 kℓ	$6 \times R29,93 = 179,58$
2	above 6 kℓ to 10 kℓ	$4 \times R52,44 = 209,76$
3	above 10 kℓ up to 20 kℓ	$10 \times R114,00 = 1\,140,00$
4	above 20 kℓ up to 35 kℓ	$3 \times R134,00 = 402,00$
5	above 35 kℓ up to 50 kℓ	
6	more than 50 kℓ	
Total: 23		R1 931,34

Question 1

Mpho lives in Johannesburg. Johannesburg water uses an increasing block tariff system for the water service. This divides water usage into blocks, where the tariff per kilolitre increases with increased consumption.

A fixed levy of R31,08 (VAT inclusive) is charged to all residents for each waterconnection.

Mpho must pay the fixed levy and for his water usage as per kilolitre according to the block tariff per kilolitre.

TABLE 1 shows the block water tariffs for Johannesburg.

TABLE 1: WATER TARIFFS OF JOHANNESBURG

Kiloliters per month	Tariff (R/kP)
0 to 6	Free
More than 6 to 10	22,26
More than 10 to 15	23,23
More than 15 to 20	32,57
More than 20 to 30	45,01
More than 30 to 40	49,23
More than 40 to 50	62,11
More than 50	66,56

(All tariffs are VAT exclusive)

[Adapted from www.joburg.org.za]

TARIFF SYSTEMS

Use the information above to answer questions that follow.

- 1.1 Mpho used 14 kilolitres of water for the month. Calculate what his monthly cost of water will be, including VAT.
- 1.2 Determine what percentage Mpho had to pay for his fixed levy on his total monthly cost for 14 kilolitres.

Solutions

$$\begin{aligned}
 1.1 \quad & \text{Cost VAT excluded/Koste BTW uitgesluit} \\
 & = (6\text{kl} \times R0) + (4\text{kl} \times R22,26) + (4\text{kl} \times R23,23) \\
 & = R181,96
 \end{aligned}$$

$$\begin{aligned}
 & \text{Cost VAT included/Koste BTW ingesluit} \\
 & = R181,96 \times \frac{115}{100} \\
 & = R209,25
 \end{aligned}$$

$$\begin{aligned}
 & \text{Total cost/Totale koste} \\
 & = R209,25 + R31,08 \\
 & = R240,33
 \end{aligned}$$

$$\begin{aligned}
 1.2 \quad & \text{Percentage/Persentasie} \\
 & = \frac{R31,08}{R240,33} \times 100\% \\
 & = 12,9\%
 \end{aligned}$$

NOTE THE FOLLOWING:

1. In this case electricity is charged according to a sliding scale.
 2. Use the same procedure as explain in water tariffs.
 3. Use the table to write down the units and calculations per block.
 4. Convert tariffs or total cost in cents to rand by **dividing by 100** (Reason: There's 100 cents in 1 Rand)
- Etc.
Convert 106,56c to rand
= $106,56 \div 100 = R1,0656$

EXAMPLE 2

ELECTRICITY TARIFFS:

1. Electricity is charged per kWh (KiloWatt – hour) in cents or rand.
2. Electricity can also be charged according to a **sliding scale** or it can be charged at a **flat rate**.
3. If charged according to a **sliding scale**, use the same procedure as **explained in water tariffs**.
4. There's currently two payment systems for electricity, namely pre-paid and on a contract basis.

Electricity purchase blocks for 20 Amp Traiffs		Tariff (cent / kWh)	
		2017	2018
Block 1	0 – 350 kWh	104.26	106.56
Block 2	More than 350 kWh (>350)	118.00	120.60

1. Calculate in Rand the electricity costs for the following monthly consumption:

- 1.1 140 kWh in 2018
- 1.2 380 kWh in 2017
- 1.3 Calculate the percentage increase in electricity charges for Block 2 tariffs from 2017 to 2018.

TARIFF SYSTEMS

1.1

Electricity purchase blocks for 20 Amp Traiffs		Tariff (cent / kWh)
		2018
Block 1	0 – 350 kWh	$140 \times 106.56 = 14\,918,4\text{ c}$ $= R149,18$
Block 2	More than 350 kWh (>350)	

1.2

Electricity purchase blocks for 20 Amp Traiffs		Tariff (cent / kWh)
		2017
Block 1	0 – 350 kWh	$350 \times 104.26 = 36491\text{c}$ $= R364,91$
Block 2	More than 350 kWh (>350)	$30 \times 118.00 = 3\,540$ $= R 35,40$

Total: **380** **R400,31**

$$\begin{aligned}
 1.3 \quad \text{Percentage increase} &= \frac{\text{New}-\text{Old}}{\text{Old}} \times 100\% \\
 &= \frac{120,60-118,00}{118,00} \times 100\% \\
 &\approx 2,2\%
 \end{aligned}$$

Question 2: This is an example of a flat rate.

2. A local municipality charges 124,5c / kWh (VAT inclusive) for pre-paid electricity.

Calculate how many units in kWh a household will get if R500 of pre-paid electricity is purchased.

ANSWER

Pre-paid charges in rand = R1,245


Number of units = $R500 \div R1,245$

$\approx 401,606$ units

TARIFF SYSTEMS

Question 2

Dream Big High School received an electricity account statement for the school from uMlalazi Municipality. ANNEXURE A is an extract of the account statement with some values and amounts left out.

UMLALAZI MUNICIPALITY			ACCOUNT NUMBER: 000002112519			
VAT REG. No.: 4170193181			INVOICE DATE: 20220510			
P O BOX 37, ESHOWE, 3815		INDIGENT DATE: 0		STREET ADDRESS/STAND KANGELA STREET ESHOWE 81 P O BOX 37 ESHOWE		
ESHOWE		NOTIFIED DEMAND: 84				
CONTACT: ESHOWE: (035) 473 3300						
MTUNZINI: (035) 473 3460						
GINGINDLOVU: (035) 473 3470						
FAX: (035) 474 4733						
TAX INVOICE STATEMENT						
VAT REFERENCE: 0		METER READING				
METER TYPE	METER No.	OLD READING	NEW READING	CONSUMPTION	READING DATE	
kVA	E6420079	0	73	73	20220419	
kWh	K6420079	959619	974631	D	20220419	
ACCOUNT DETAILS						
DATE	CODE	DESCRIPTION	UNITS	TARIFF (in Rands) 15% VAT Inclusive	VALUE (in Rands) 15% VAT Inclusive	
20220419		OPENING BALANCE	0	.00000	36 927,84	
20220419	E311	R: ER-SG-Electricity-ES-Industry	73	228,45507	16 677,22	
20220419	E310	R: ER-SG-Electricity-ES-Industry	D	R0.91965	13 805,86	
20220419	ECAP	R: ER-SG-Electricity-ES-Industry	0	.00000	555,86	
20220419	R440	R: ER-SG-Electricity-ES-Industry	0	217.50000	2 501,25	
20220503		PAYMENT VB0620T	0	.00000	-36 927,84	
120 DAYS+	90 DAYS	60 DAYS	30 DAYS	CURRENT	VAT	TOTAL DUE
.00	.00	.00	.00	E	4 374,81	E
New tariffs implemented with effect from 01 July 2017			DUE DATE		RECEIPT UP TO	
			20220531		20220430	
Banking details: FNB, ACCOUNT NUMBER: 52191999999						
*Accounts unpaid on the date payable are subject to interest charged at a standard rate of 10% per annum, compounded monthly and services will be suspended. Please notify the municipality in writing of the termination of any services, in order to avoid being held responsible for any costs.						
<ul style="list-style-type: none"> • kWh (kilowatt hour) is the unit of three-phase electricity. • kVA (kilovolt-ampere) is the unit for transformer electricity. 						

Use ANNEXURE A to answer the following questions.

- 1.1 Explain the term “opening balance” in this context.
- 1.2 Show, using the meter readings that the value of **D** is 15 012.
- 1.3 Calculate the missing value **E**.
- 1.4 Use calculations to verify if the VAT amount of R4 374,81 was calculated correctly.

TARIFF SYSTEMS

ANSWERS

1.1 Amount owed by the school for electricity brought forward at the start of the account period.

$$\begin{aligned} 1.2 \quad D &= 974\,631 - 959\,619 \\ &= 15\,012 \end{aligned}$$

$$\begin{aligned} 1.3 \quad E &= R16\,677,22 + R13\,805,86 + R555,86 + R2\,501,25 \\ &= R33\,540,19 \end{aligned}$$

$$\begin{aligned} 1.4 \quad \text{VAT exclusive amount} &= R33\,540,19 \div 1,15 \\ &= R29\,165,38 \end{aligned}$$

OR

$$\begin{aligned} \text{VAT exclusive amount} &= R33\,540,19 \times 100 \div 115 \\ &= R29\,165,38 \end{aligned}$$

OR

$$\begin{aligned} \text{VAT exclusive amount} &= R33\,540,19 \div 115\% \\ &= R29\,165,38 \end{aligned}$$

$$\begin{aligned} \text{VAT exclusive amount} &= R33\,540,19 \times (15 \div 115) \\ &= R29\,165,38 \end{aligned}$$

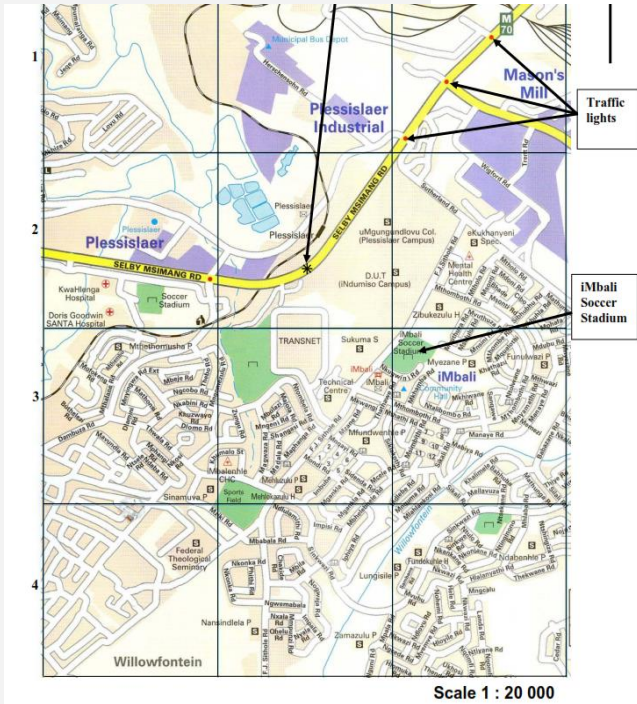
$$\begin{aligned} \text{VAT} &= R33\,540,19 - R29\,165,38 \\ &= R4\,374,81 \end{aligned}$$

YES it was calculated correctly

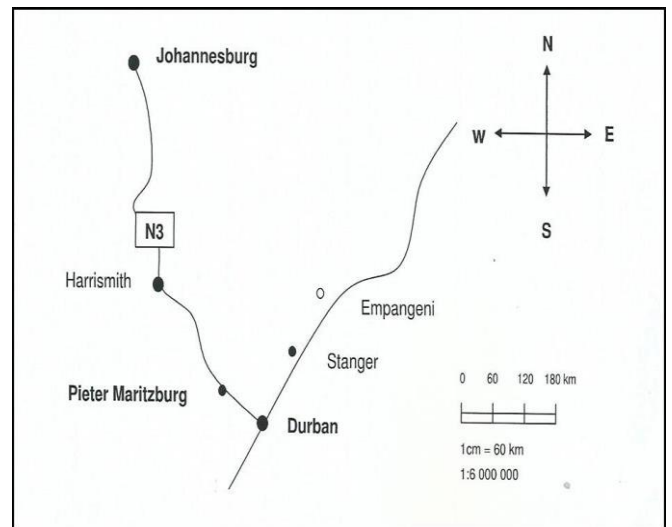
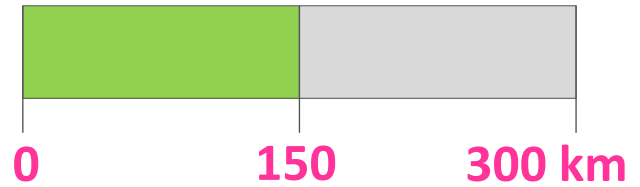
SESSION 2: SCALE

NUMBER/RATIO SCALE:

• 1 : 20 000



BAR/GRAPHIC SCALE:



NUMBER/RATIO SCALE

1

:

80

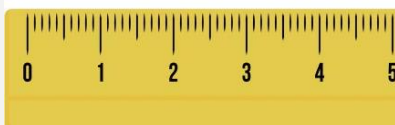
1 cm

:

80 cm

PAPER

REALITY



MEASURE!

1 refers to ONE unit on PAPER :
 What are the most appropriate units that we can measure on paper? **cm** or **mm**

80 refers to 80 units in reality or the actual length/distance. The actual units can now also be written in **cm** or **mm** depending on the **measurement units** used on **paper**

SCALE

Scale is expressed as a **ratio** and this type of scale contains a special ratio, namely a **UNIT RATIO**,
1 : ...

If we **measure in cm on PAPER** the corresponding units representing the actual length/distance will also be in **cm**.

OR

If we **measure in mm on PAPER** the corresponding units representing the actual length/distance will also be in **mm**.

1 mm

:

80 mm

The most **appropriate measurement units of lengths/distances to be used in reality** are metres or kilometres - CONVERT from **cm/mm** to m/km

CONVERT!

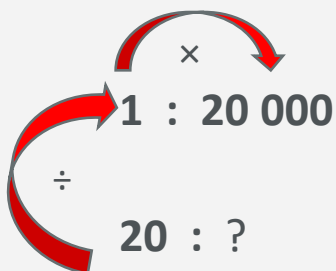
NUMBER SCALE

Using a given **number scale** to calculate real/actual length or distances:

Given scale: 1 : 20 000

If 20 cm is measured on a map, calculate the real distance in km

METHOD 1



METHOD 1: Divide up Multiply down

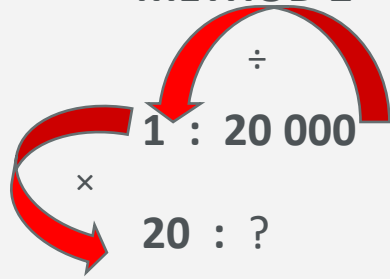
$$\begin{aligned} \text{Real distance} &= 20 \div 1 \times 20\,000 \\ &= 400\,000 \text{ cm} \end{aligned}$$

(Convert to km)

$$\begin{aligned} \text{In km} &= 400\,000 \div 100\,000 \\ &= 4 \text{ km} \end{aligned}$$

SCALE

METHOD 2



Summary:

1. If needed **MEASURE lengths** on paper
2. Write down the scale.
3. Write the corresponding lengths beneath each other.
4. Apply one of the two methods or the method that you're used to or know.
5. Convert to appropriate/unit requested. Know your conversions!

METHOD 2: Multiply down

$$\begin{aligned} \text{Real distance} &= 20\ 000 \times 20 \\ &= 400\ 000 \text{ cm} \end{aligned}$$

(Convert to km)

$$\begin{aligned} \text{In km} &= 400\ 000 \div 100\ 000 \\ &= 4 \text{ km} \end{aligned}$$

QUESTION 1

The length of one side of a building is 9,5 m. The length on the plan of the same side is 4,75 cm. Determine the scale of the plan in the form, 1 : ...



SOLUTION:

PAPER

ACTUAL LENGTH/DISTANCE

$$4,75 \text{ cm} = 9,5 \text{ m} \quad (\text{Step 1: Write down in scale form/order; equate the corresponding lengths})$$

$$4,75 \text{ cm} = 950 \text{ cm} \quad (\text{Step 2: Convert to the same units})$$

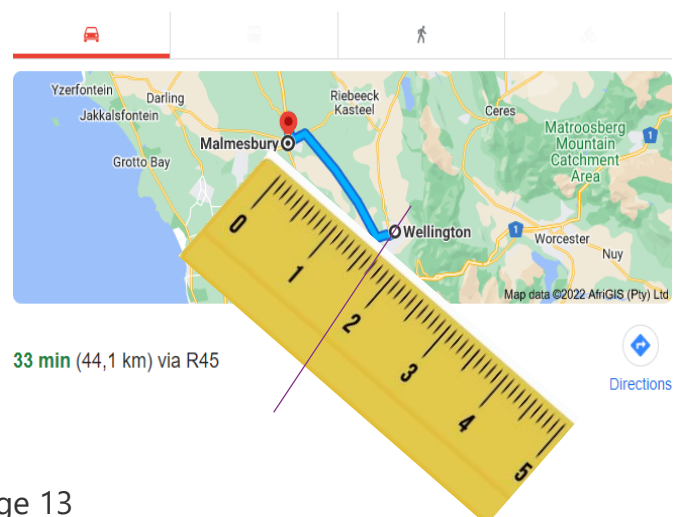
$$4,75 \div 4,75 = 950 \div 4,75 \quad (\text{Step 3: Divide both sides by plans length})$$

$$1 \text{ cm} : 200 \text{ cm} \quad (\text{in order to get } 1 : \dots)$$

$$1 : 200 \quad (\text{Step 4: Writing without units, to get the unit ratio/scale})$$

QUESTION 2

The distance between Wellington and Malmesbury is 44,1 km. The length on the plan between the towns is 18 mm. Determine the scale of the plan in the form, 1 : ...



SCALE

SOLUTION:

PAPER	ACTUAL LENGTH/DISTANCE
-------	------------------------

$$18 \text{ mm} = 44,1 \text{ km} \quad (\text{Step 1: Write down in scale form/ order; equate the corresponding lengths})$$

$$0,018 \text{ m} = 44 \text{ 100 m} \quad (\text{Step 2: Convert to the same units})$$

$$0,018 \div 0,018 = 44 \text{ 100} \div 0,018 \quad (\text{Step 3: Divide both sides plans/maps length in order to get } 1 : \dots)$$

$$1 \text{ m} : 2 \text{ 450 000 m}$$

$$1 : 2 \text{ 450 000} \quad (\text{Step 4: Writing without units, to get the unit ratio/scale})$$

QUESTION 3

Given a scale of 1 : 250. The measured length is 10 cm. Calculate the real/actual length in metre.

Solution

$$1 : 250$$

$$10 \text{ cm} : \dots$$

$$1 : 250 \times 10 \text{ cm}$$

$$2 \text{ 500 cm}$$

Actual length in m

$$= 2 \text{ 500} \div 100$$

$$= 25 \text{ m}$$

QUESTION 4

If the measured length is 280 mm and the actual distance is 15 km, determine the scale of the map in the form of 1 :

Round your answer to the nearest thousand.

Solution

Measured length : Actual length

$$280\text{mm} : 15 \text{ km}$$

(Convert mm and km to cm)

$$28 \text{ cm} : 15 \text{ km} \times 100 \text{ 000}$$

$$28 \text{ cm} : 1 \text{ 500 000 cm}$$

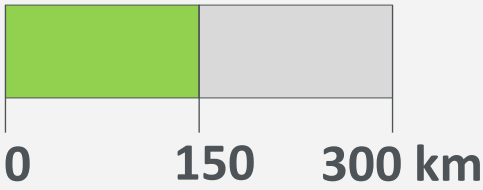
$$28 \div 28 : 1 \text{ 500 000} \div 28$$

$$1 : 53 \text{ 571,4}$$

$$1 : 53 \text{ 000}$$

SCALE

• BAR SCALE:



• 1. MEASURE!

• 2. MEASURE!

EXAMPLE

If the length on the map between two towns is 4 cm, calculate the actual distance in km between the towns.

SOLUTION:

PLAN

ACTUAL

$$2,5 \text{ cm} \times = 150 \text{ km} \quad (\text{Step 1: Measure the bar scale})$$

$$4 \text{ cm} = \text{Distance in km} \quad (\text{Step 2: Write down the measured length of the two towns on the map beneath each other})$$

$$\text{Distance} \times 2,5 \text{ cm} = 150 \text{ km} \times 4 \text{ cm} \quad (\text{Step 3 Cross multiplication})$$

$$= 150 \text{ km} \times 4 \text{ cm} \div 2,5 \text{ cm}$$

$$= 240 \text{ km}$$

SOLUTION:

PLAN

ACTUAL

$$2,5 \text{ cm} \times = 150 \text{ km} \quad (\text{Step 1: Measure the bar scale})$$

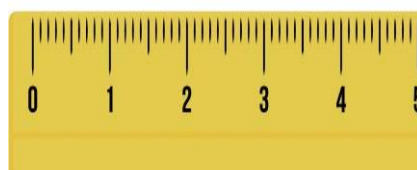
$$4 \text{ cm} = \text{Distance in km} \quad (\text{Step 2: Write down the measured length of the two towns on the map beneath each other})$$

$$\text{Distance} = 4 \text{ cm} \div 2,5 \text{ cm} \times 150 \text{ km} \quad (\text{Step 3: Divide Up multiply across})$$

$$= 240 \text{ km}$$

Question 1

Given the bar scale below: If the measured length is 15 cm calculate the real/actual distance in km.



SCALE

SOLUTION

$$2 \text{ cm} : 150 \text{ km}$$

$$15 \text{ cm} : \dots$$

$$2 \text{ cm} : 150 \times 100\,000$$

$$\mathbf{15\,000\,000 \text{ cm}}$$

$$15 \text{ cm} : 15 \div 2 \times 15\,000\,000$$

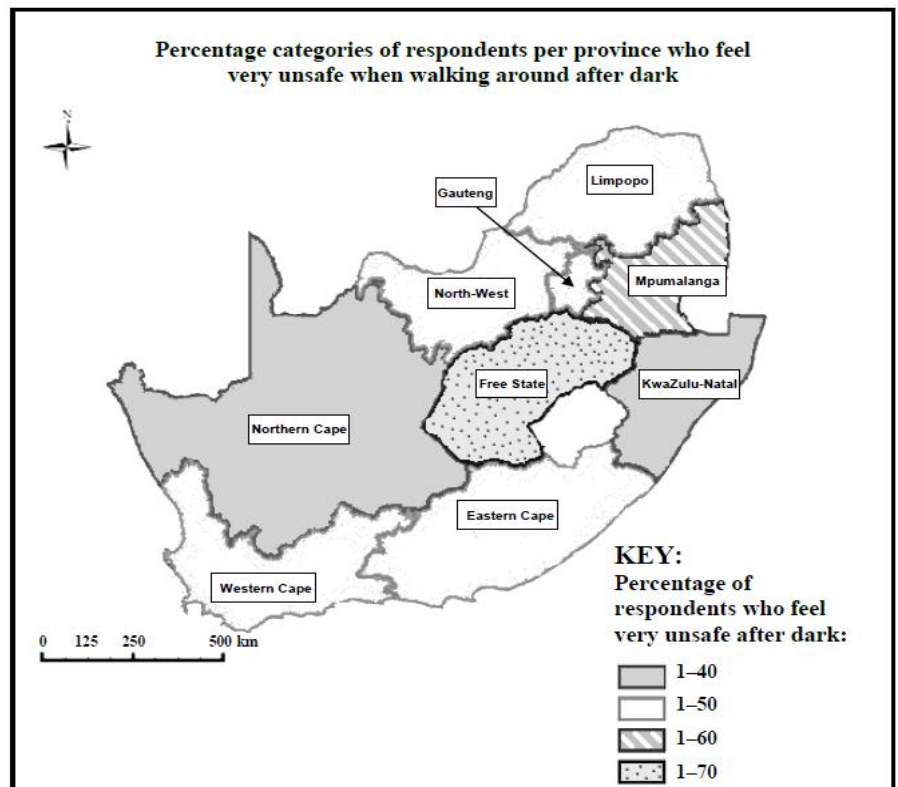
$$112\,500\,000 \text{ cm}$$

Actual length (Convert to km)

$$= 112\,500\,000 \div 100\,000$$

$$= \mathbf{1\,125 \text{ km}}$$

Question 2



2.1 In which provinces(s) did 40% and less of the respondents feel very unsafe when walking around after dark.

2.2 In which percentage category do the majority of the provinces fall?

SCALE

2.3 Which province is south-west of Free State and at the same time south of the Northern Cape.

2.4 Calculate the scale used on the map in the form 1 : ...

SOLUTIONS

2.1 Northern Cape

Kwazulu-Natal

2.2 41 – 50%

2.3 Western Cape

2.4 Measured : Actual length

2 cm : 500 km

2 cm : 500 km \times 100 000

2 cm : 50 000 000 cm

2 \div 2 : 50 000 000 \div 2

1 : 25 000 000