

2023 SUBJECT WORKBOOK

Grade 12

$a+b=c$ MATHEMATICAL LITERACY

A joint initiative between the Western Cape Education Department and Stellenbosch University.

BROADCAST SESSIONS

GRADE 12

MEASUREMENT

GRADE 12

MEASUREMENT

Session	Date	Time	Topic
1	26/04/2023	15h00-16h00	Measurement – perimeter and area
2	15/05/2023	15h00-16h00	Measurement - Volume

INTRODUCTION AND TOPICS

Perimeter and Area

Definition Perimeter

- The border or outer boundary lengths of a two-dimensional figure
- The perimeter of a circle is known as the circumference

Definition Area

- The surface enclosed by the boundary lengths of a two-dimensional figure

Volume

It is important to be able to visualize the different shapes in everyday objects. In this case we need to know which polygon is represented by which object.

The formulae are generally provided, so the important thing here, is to be able to substitute values correctly into the formulae.

Topics	Description
Perimeter and Area	<p>When we think of perimeter, we usually think of 2-dimensional, flat shapes, but it can also refer to 3-dimensional shapes that consist of a framework.</p> <p>Area is the total portion that falls within the perimeter.</p>
Volume	<p>Volume refer to 3-dimensional shapes</p> <p>Understand and use appropriate vocabulary, such as: equation, formulae, perimeter, radius, diameter, length, breadth, height, base, circumference, volume, circle, cylinder, polygons, right prisms, triangular, rectangular and square.</p>

TERMINOLOGY

Term	Definition
Area	The amount of two-dimensional space occupied by a 2-D shape. The area of a shape is the size of its surface.
Circumference	Distance around a circle / the perimeter of a circle.
Diameter	A straight line passing through the centre of a circle and touching the circle at both ends, thus dividing the circle into two equal halves.
Dimension	A measurable extent, e.g. length, breadth, height, depth, time. Physics, technical: the base units that make up a quantity, e.g. mass (kg), distance (m), time (s).
Perimeter	The total distance around the boundary or edge that outlines a specific shape.
Radius	The distance from the centre of the circle to any point on the circumference of the circle.
Cylinder	A 3-dimensional object with congruent parallel sides and bases are circles. A tall shape with parallel sides and a circular cross-section – think of a log of wood, for example, or a tube.
Surface Area	The area of all the faces / surfaces of an object added together.
Volume	The amount of 3-D space occupied by an object. It is measured in cubic units

SESSION 1 | PERIMETER AND AREA

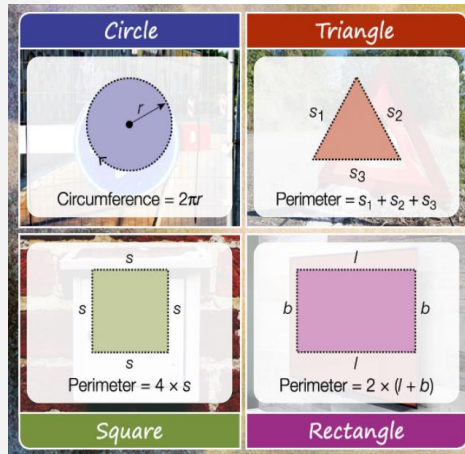


WHAT YOU SHOULD KNOW

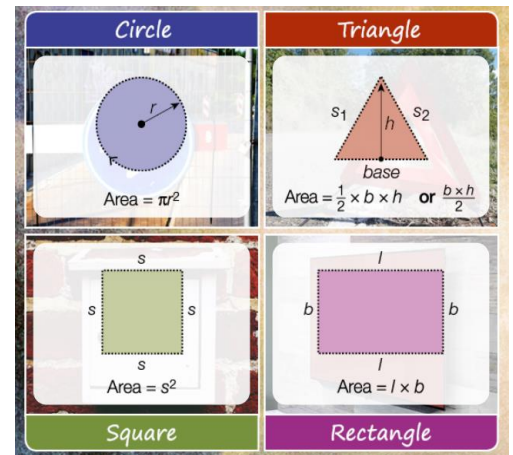
All formulae for calculations involving area will be provided in assessments

You must just select the correct formulae and substitute.

Perimeter



Area



Example 1

Your Mathematical Literacy classroom gets new tables, shaped as shown on the left.

- 1.1 Using the appropriate formulae, calculate the area of the table, in m^2 .
- 1.2 If each table cost R615 and ten tables were bought, calculate how much the tables cost per m^2 .

Solution:

- 1.1 We can see that the table is made up of two identical triangles, and one rectangle.

The formula for the area of a triangle is:

$$\frac{1}{2} \times \text{base} \times \text{height}$$

So, the area of one of our triangles is:

$$\frac{1}{2} \times 500\text{mm} \times 70\text{cm}$$

$$\frac{1}{2} \times 0,5\text{m} \times 0,7\text{m} \quad (\text{change the units to metres})$$

$$= 0,175 \text{ m}^2$$

The formula for the area of a rectangle is: length \times breadth.

$$\text{So, the area of the middle rectangle is} = 0,9 \text{ m} \times 70 \text{ cm}$$

$$= 0,9 \text{ m} \times 0,7 \text{ m} \quad (\text{change the units to metres})$$

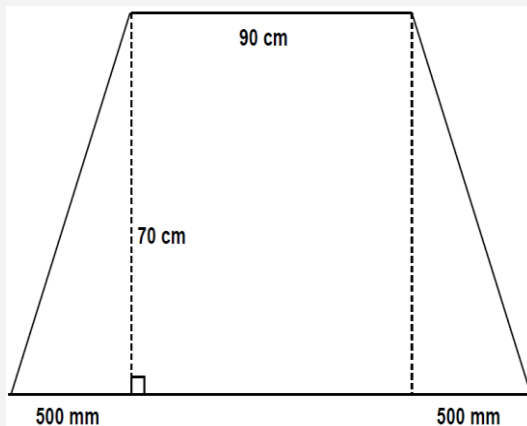
$$= 0,63 \text{ m}^2$$

Now we simply add the three areas together:

$$\text{Area triangle} + \text{area rectangle} + \text{area triangle}$$

$$= 0,175 \text{ m}^2 + 0,63 \text{ m}^2 + 0,175 \text{ m}^2$$

$$= 0,98 \text{ m}^2$$



PERIMETER AND AREA

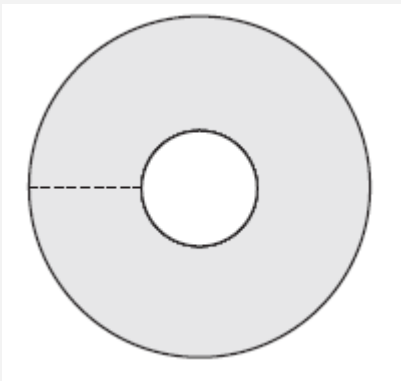
- 1.2 10 tables will cost $R615 \times 10 = R6\ 150$.
- 10 tables will have a total area of $0,98\ m^2 \times 10 = 9,80\ m^2$.
- $R6\ 150 \div 9,80\ m^2 = R627,55$
- So, the tables cost $R627,55$ per square metre.

OR

$$R615 \div 0,98 = R627,55$$

Example 2

Mrs. Dlamini buys a new lampshade for a lamp. She measures the radius of the inside circle in the lampshade to be 50 mm. The diameter of the outside (larger) circle is 40 cm. (Note, the diagram is not drawn to scale.)



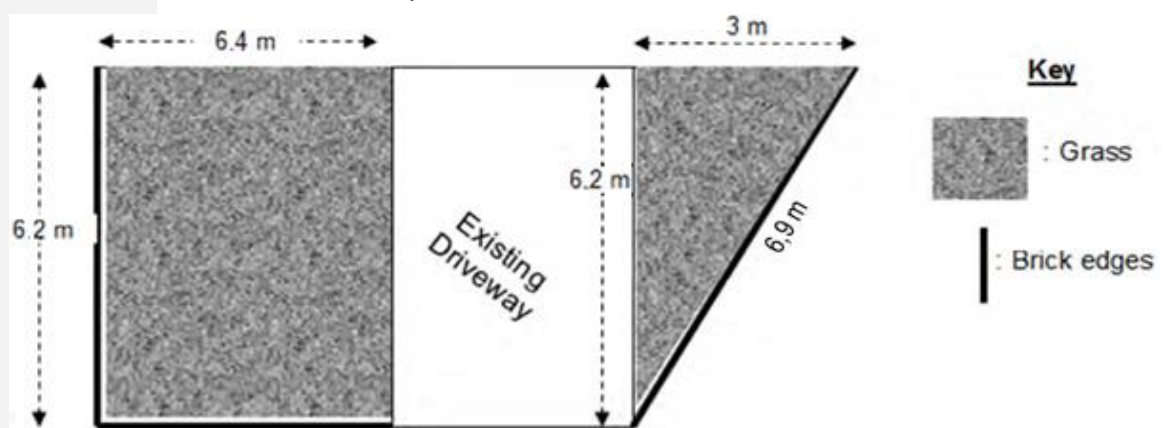
- 2.1 Calculate the circumference of the smaller, inner circle (in cm).
- 2.2 Calculate the circumference of the larger, outer circle (in cm). Round off your answer to one decimal place.
- 2.3 Calculate the perimeter of half of the larger, outer circle (in cm).
- 2.4 Calculate the width of the area shown by the dotted line in the diagram above.

Solution

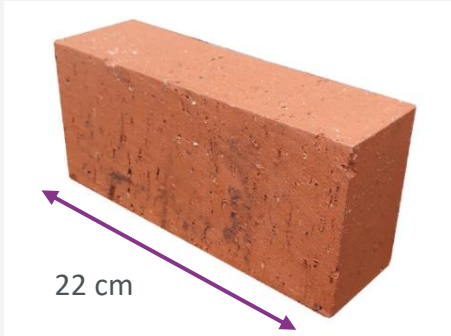
- 2.1 Inside circle perimeter/circumference = $2 \pi r = 2 \times 3,142 \times 5\ cm$
 $= 31,42\ cm$
- 2.2 Circumference/perimeter = $2 \pi r = 2 \times 3,142 \times 20\ cm = 125,7\ cm$
- 2.3 Half perimeter = $\frac{\text{Perimeter}}{2} = \frac{125,7}{2} = 62,85\ cm$
- 2.4 Inner circle radius = 5 cm. Entire radius = 20 cm.
 Difference between radii = $20\ cm - 5\ cm = 15\ cm$

Exercise

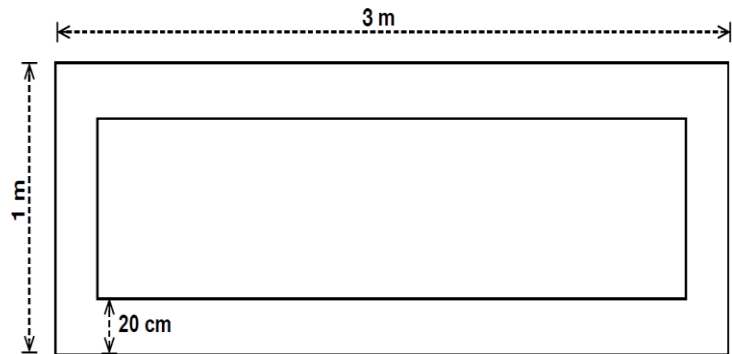
1. A man wants to extend his driveway to the sides to create some new parking areas. He is going to remove the grass that is already there and put down stones.



PERIMETER AND AREA



- 1.1 Firstly, he removes the grass and replant it in other areas of his yard. Calculate the total area of grass that will be removed from both areas. You may choose to use some of the formulas above.
- 1.2 To keep the stones in position he is going to lay some bricks along the edges marked in bold. The other edges are next to the house or bordering on the existing driveway and will not require bricks.
 - 1.2.1 Calculate the total length of all the edges where he is going to lay bricks.
 - 1.2.2 Each brick is 22 cm long. How many bricks is he going to lay along the given edges?
2. Jenny has started a decorating business and has a contract to provide decor at a wedding reception.



- 2.1 The tables used at this wedding are rectangular with a length of 3 m and a width of 1 m as shown above. The fabric she plans to use for the tablecloth costs R75 per metre (but can be bought in lengths smaller than a metre) and is sold in rolls that are 1,4 m wide. The bride and groom want the tablecloths to hang at least 20 cm over the edges of the tables.

Calculate the cost of the cloth for each table.

- 2.2 If there are 15 tables at the wedding, calculate how much she is going to spend on tablecloths alone.



3. You build an extra room onto your house. You want to cover the roof with tiles. Overlap where adjacent tiles will be joined is 75 mm. The overlap in the length will be a third of the tile (effective length = $\frac{2 \times \text{length}}{3}$).
 - 3.1 How many tiles do we need to cover a roof that is 3 m wide and 2 m long.
 - 3.2 Calculate the total cost of the tiles (VAT inclusive), if the prize of one tile is R10,25 VAT exclusive.

PERIMETER AND AREA

Solutions

$$1.1 \quad \text{Rectangular area} = 6,4 \text{ m} \times 6,2 \text{ m} = 39,68 \text{ m}^2$$

$$\text{Triangular area} = \frac{1}{2} \times 6,2 \text{ m} \times 3 \text{ m} = 9,3 \text{ m}^2$$

$$\text{Total area} = 39,68 \text{ m}^2 + 9,3 \text{ m}^2 = 48,98 \text{ m}^2$$

$$1.2.1 \quad \text{Edge of the triangle is the hypotenuse of a right-angled triangle:}$$

$$\text{Edges of rectangle} = 6,4 \text{ m} + 6,2 \text{ m} = 12,6 \text{ m}$$

$$\text{Total length to be covered} = 12,6 \text{ m} + 6,9 \text{ m} = 19,5 \text{ m}$$

$$1.2.2 \quad \text{One brick} = 22 \text{ cm} = 0,22 \text{ m}$$

$$\text{No. of bricks} = 19,5 \text{ m} \div 0,22 \text{ m} = 88,64 \text{ bricks} \approx 90 \text{ bricks}$$

$$2.1 \quad (3,0 + 0,2 + 0,2) \times R75 \\ = (3,4 \times 75) \\ = R255,00$$

$$2.2 \quad 15 \times R255 \\ = R3\,825$$

$$3.1 \quad \text{Width of a tile} = 33 \text{ cm} - 7,5 \text{ cm} = 25,5 \text{ (part of the tile that you can see)}$$

$$\text{Total width} = (25,5 \times a) + 7,5 = 300 \text{ cm}$$

$$\therefore 25,5a = 300 - 7,5 = 292,5$$

$$25,5a = 292,5$$

$$a = 11,4 \approx 12$$

$$\text{Height of a tile} = \frac{2}{3} \times 42 = 28 \text{ cm (part of the tile that you can see)}$$

$$\text{Total height} = (28 \times b) + 14 = 200 \text{ cm}$$

$$\therefore 28b = 200 - 14 = 186$$

$$28b = 186$$

$$b = 6,64 \approx 7$$

$$\text{Total number of tiles} = 12 \times 7 = 84$$

$$3.2 \quad \text{Cost excluding VAT} = 84 \times R10,25 \\ = R861$$

$$\text{Total cost including VAT} = R861 \times 1,15 \\ = R990,15$$

SESSION 2 | VOLUME

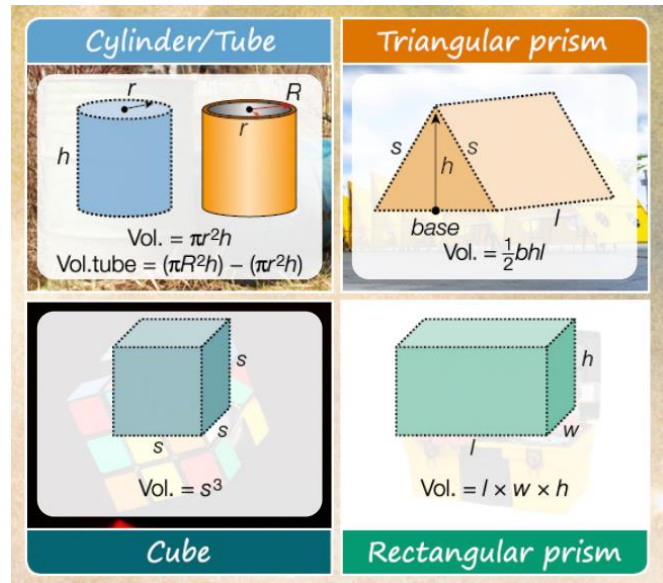


WHAT YOU SHOULD KNOW

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Volume



Example 1

The rectangular long jump pit at the school is 2,75 m wide and 9 m long and is filled with sand.

- 1.1 Calculate the volume of sand needed to fill the long jump pit to a depth of 70 mm. Give the answer rounded off to THREE decimal places. Use the formula: **Volume = length × breadth × height**
- 1.2 Would you say that the sand used is enough to ensure that learners do not get hurt? Explain.

Solution:

1.1 70 mm = 0,07 m

Volume = length × breadth × height

$$= 2,75 \times 9 \times 0,07$$

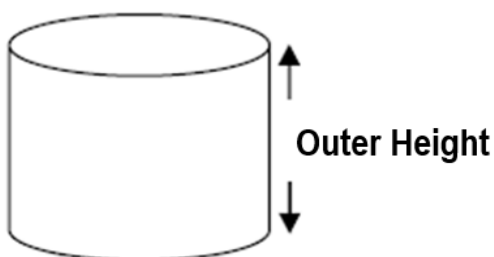
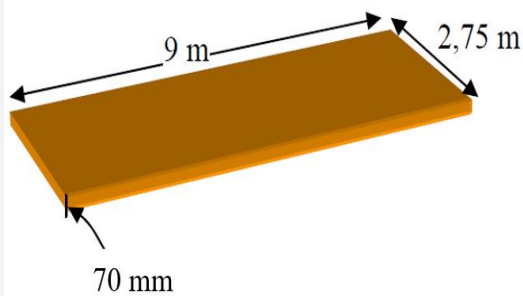
$$= 1,7325 \text{ m}^3$$

- 1.2 No, 70 mm is not even one third of a ruler, so learners would land on a very hard surface, that could result in injury.

Example 2

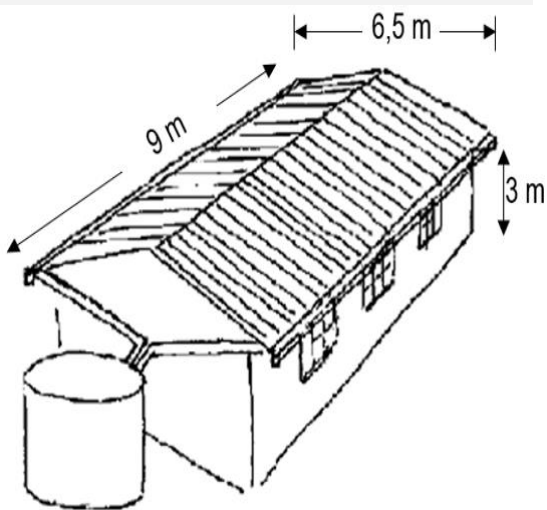
Jabu Ndou requires a cylindrical water tank to collect rainwater from his roof. This water will be used for irrigating his garden.

Jabu wants to know how much rainwater the tank can hold. The inner radius of the tank is 0,998 m and the inner height of the tank is 2,498 m.



Cylindrical rainwater tank

VOLUME



2.1 Calculate the total volume, rounded off to THREE decimal places, of the water tank.

Use the formula:

Volume of a cylinder = $\pi \times (\text{radius})^2 \times \text{height}$, and using $\pi = 3,142$

2.2 Determine the height, rounded off to THREE decimal places, of the water in the tank when it is 80% full.

Solution

2.1 **Volume of cylinder** = $\pi \times (\text{radius})^2 \times \text{hoogte}$

$$= 3,142 \times 0,998^2 \times 2,498$$

$$= 7,812 \text{ m}^3$$

2.2 Height = $\frac{80}{100} \times \frac{2,498}{1}$

$$= 1,998 \text{ m}$$

$$= 7,812 \text{ m}^3$$

Exercise

1. A school builds a swimming pool with the following dimensions of, length = 15 m; depth = 1,3 m to the filling level, and width = 5 m. (NOTE: $1 \text{ m}^3 = 1\,000 \text{ liter}$ and $1\,000 \text{ liter} = 1 \text{ kl}$)

1.1 Calculate the volume of the swimming pool up to the level it is filled.

1.2 Convert this volume (a) to litres (b) and kilolitres.

1.3 When the school fills the pool, they use a pump which pumps water at a rate of 2 liter per second. How long would it take to fill up the pool? Give your answer in hours and minutes.

1.4 Water costs R8,64 per kiloliter. How much will it cost the school to fill up the pool?

1.5 If after the children were splashing in the pool and the water level dropped to 1,24 meters, how much water, in liters, do they need to top it up with?

2. Wandile decides to grow his own vegetables. He makes a rectangular vegetable garden with length = 2,5 m and breadth = 1,5 m.

2.1 Wandile adds a 7,5 cm layer of compost to his vegetable garden.

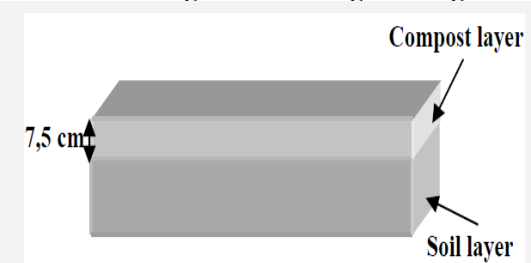
Calculate the volume of the compost added in cm.

Use the formula:

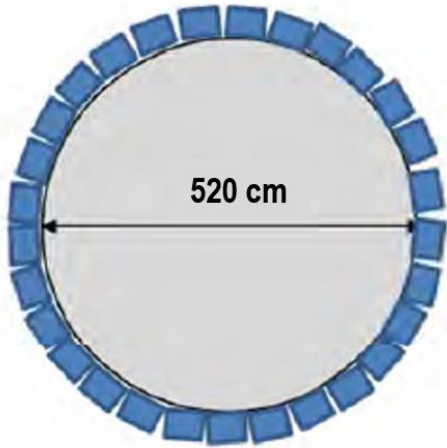
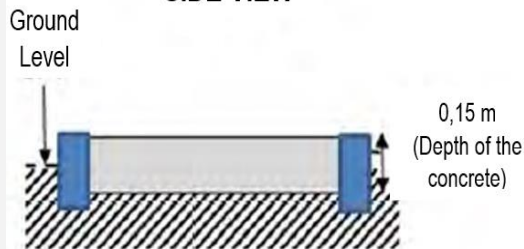
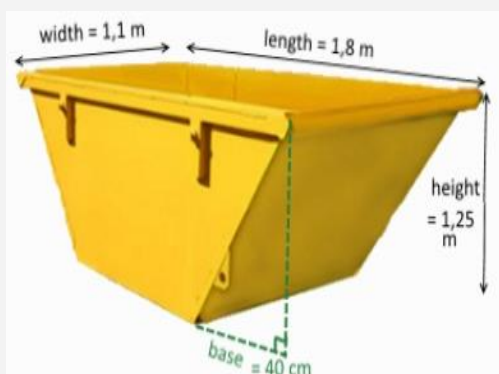
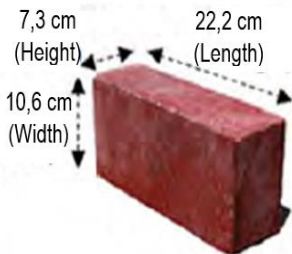
Volume of a rectangular prism = length \times breadth \times height



Shade-netting over the vegetable garden



VOLUME

TOP VIEW

SIDE VIEW

Dimensions of a brick


2.2 If the total volume of the compost and the soil layer is $843\,750\text{ cm}^3$, determine the height of the soil without the compost.

3. A circular concrete slab is planned for an outdoor picnic area. The circular concrete slab will be surrounded by bricks along the perimeter as follows: (NOTE: Drawings are NOT to scale)

3.1 Convert 520 cm to metres.

3.2 Use the following equation to calculate the volume of the concrete in the circular concrete slab. Answer in m^3 . ($\pi = 3,142$)

$$\text{Volume} = \pi \times \text{radius}^2 \times \text{height}$$

3.3 Calculate the number of bricks you will need to fit next to each other, if it is placed side to side along the length of the brick.

You may use the following formula:

$$\text{Perimeter} = 2 \times \pi \times \text{radius}, \text{ where } \pi = 3,142$$

4. Determine the volume of sand that can be dumped in the skip shown on the bottom left.

Volume of a rectangular prism = length \times breadth \times height

Volume of triangular prism = $\frac{1}{2} \times$ base \times height \times width

Solutions

1.1 $15\text{ m} \times 5\text{ m} \times 1,3\text{ m}$
 $= 97,5\text{ m}^3$

1.2 (a) 97 500 liters
 (b) 97,5 kl

1.3 Time to fill up = $\frac{97500}{2}$
 $= 48750\text{ seconds}$ ($\div 60$)
 $= 812,5\text{ minutes}$ ($\div 60$)
 $= 13,54166667\text{ hours}$
 $\therefore 0,5416667 (\times 60)$
 $= 32,5\text{ minutes}$
 So, the total time taken is 13 hours 32½ min

1.4 $R8,64 \times 97,5\text{ kl} = R842,40$

1.5 $1,3\text{ m} - 1,24\text{ m} = 0,06\text{ m}$
 $\therefore 15\text{ m} \times 5\text{ m} \times 0,06\text{ m}$
 $= 4,5\text{ m}^3$
 $= 4500\text{ liters}$

VOLUME

2.1 $2,5 \text{ m} = 250 \text{ cm}$

$1,5 \text{ m} = 150 \text{ cm}$

Volume of a rectangular prism = length \times breadth \times height

$$= 250 \text{ cm} \times 150 \text{ cm} \times 7,5 \text{ cm}$$

$$= 281\,250 \text{ cm}^3$$

2.2 **Volume of a rectangular prism = length \times breadth \times height**

$$843\,750 \text{ cm}^3 = 250 \text{ cm} \times 150 \text{ cm} \times \text{height with compost}$$

$$\frac{843\,750}{37\,500} = \text{height with compost}$$

$$22,5 \text{ cm} = \text{height with compost}$$

$$\text{Therefore, height of soil} = 22,5 \text{ cm} - 7,5$$

$$= 15 \text{ cm}$$

3.1 $520 \text{ cm} = 5,2 \text{ m}$

3.2 $\text{Vol} = 3,142 \times (2,6 \text{ m})^2 \times 0,15 \text{ m}$

$$= 3,142 \times (6,76 \text{ m}^2) \times 0,15 \text{ m}$$

$$= 3,19 \text{ m}^3$$

3.3 $\text{Perimeter} = 2 \times \pi \times \text{radius}$

$$= 2 \times 3,142 \times 2,6 \text{ m}$$

$$= 16,3384 \text{ m}$$

$$16,3384 \text{ m} = 1633,84 \text{ cm}$$

$$\text{Number of bricks} = 1633,84 \text{ cm} \div 10,6 \text{ cm}$$

$$= 154 \text{ bricks}$$

4. Divide the object into regular shapes:

The length of the rectangular part

$$= 1,8 \text{ m} - 2 \times \text{base of triangular sections}$$

$$= 1,8 \text{ m} - (2 \times 0,4)$$

$$= 1 \text{ m}$$

Total volume

$$= (2 \times \text{volume of triangular prism}) + \text{volume of rectangular prism}$$

$$= (2 \times \frac{1}{2} \times 0,4 \times 1,25 \times 1,1) + (1 \times 1,25 \times 1,1)$$

$$= 1,925 \text{ m}^3$$

