Western Cape Government

Education

Western Cape Education Department

Telematics
Learning Resource 2016

MATHEMATICS
Grade 12

Dear Grade 12 Learner

In 2016 there will be 6 Telematics sessions, 2 sessions per term. This workbook provides the activities for these sessions. Please make sure that you bring this workbook along to each and every Telematics session.
In term one the presenters will revise functions, inverse function and the log graph as the inverse of the exponential graph. Please ensure that you revise all the graphs done in grade 11 before these sessions start. In the grade 12 examination this section of the graphs will be $\pm$ 35 marks of the 150 marks of Paper 1.

In term 2 trigonometry is revised with the focus on Compound and Double angles. Before this session please ensure that you revise the Trigonometry done in grade 11. Differential Calculas with specific focus on The cubic graph is done in the 4th Telematics Session.

The lessons in Term 3 will focus on revision of Grade11 and Grade 12 geometry. The Grade 11 geometry entails the circle geometry theorems dealing with angles in a circle, cyclic quadrilaterals and tangents. The Grade 12 geometry is based on ratio and proportion as well as similar triangles. Grade 11 geometry is especially important in order to do the grade 12 Geometry hence this work must be thoroughly understood and regularly practiced to acquire the necessary skills.

Your teacher should indicate to you exactly which theorems you have to study for examination purposes. There are altogether 6 proofs of theorems you must know because it could be examined. These theorems are also marked with (**) in this Telematics workbook, 4 are grade 11 theorems and 2 are grade 12 theorems.

At the start of each lesson, the presenters will provide you with a summary of the important concepts and together with you will work though the activities. You are encouraged to come prepared, have a pen and enough paper (ideally a hard cover exercise book) and your scientific calculator with you.

You are also encouraged to participate fully in each lesson by asking questions and working out the exercises, and where you are asked to do so, sms or e-mail your answers to the studio.

Remember:" Success is not an event, it is the result of regular and consistent hard work".

GOODLUCK, Wishing you all the success you deserve!

Term 1: February and March (Grade 12)				
Day	**Date**	**Time**	**Subject**	**Topic**
Tuesday	9 February	15:00 – 16:00	Mathematics	Functions & Inverse Functions
Thursday	3 March	15:00 – 16:00	Mathematics	The log and exponential functions as inverses of each other

Term 2: April and May (Grade 12)				
Day	**Date**	**Time**	**Subject**	**Topic**
Tuesday	12 April	15:00 – 16:00	Mathematics	Trigonometry
Tuesday	17 May	15:00 – 16:00	Mathematics	Calculus

Term 3: July, August and September (Grade 12)				
Day	**Date**	**Time**	**Subject**	**Topic**
Wednesday	27 July	15:00 – 16:00	Mathematics	Geometry
Tuesday	23 August	15:00 – 16:00	Mathematics	Geometry

Session 1: The concept of an inverse; the inverses of $y = mx + c$ and $y = ax^2$

An inverse function is a function which does the "reverse" of a given function. More formally, if f is a function with domain X, then f^{-1} is its inverse function if and only if $f^{-1}(f(x)) = x$ for every $x \in X$.

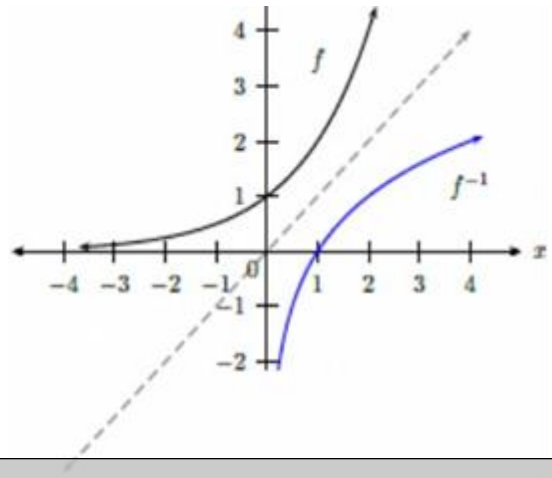
A function must be one-to-one relation if its inverse is to be a function. If a function f has an inverse f^{-1}, then f is said to be invertible.

Given the function $f(x)$, we determine the inverse $f^{-1}(x)$ by:

- Interchanging x and y in an equation;
- Making y the subject of the equation;
- Expressing the new equation in function notation.

Note:
If the inverse is not a function then it cannot be written in function notation. For example, the inverse of $f(x) = 3x^2$ **cannot** be written as $f^{-1}(x) = \pm\sqrt{\frac{1}{3}x}$ as it is not a function. We write the inverse as $y = \pm\sqrt{\frac{1}{3}x}$ and conclude that $f(x) = 3x^2$ is not invertible.

If we represented the function f and the inverse f^{-1} graphically, the two graphs are reflected about the line $y = x$. Any point on the line $y = x$ has x- and y- coordinates with the same numerical value, for example (-3; -3) and $(\frac{4}{5};\frac{4}{5})$. Therefor interchanging the x- and y- coordinates makes no difference. Below is an example to illustrate this:



Important: for f^{-1}, the superscript -1 is not an exponent. It is the notation for indicating the inverse of a function. Do not confuse this with exponents, such as $(\frac{1}{2})^{-1}$ or $3 + x^{-1}$.

Be careful not to confuse the inverse of a function and the reciprocal of a function:

Inverse	**Reciprocal**
$f^{-1}(x)$	$\lvert f(x)\rvert^{-1} = \frac{1}{f(x)}$
$f(x)$ and $f^{-1}(x)$ are symmetrical about $y = x$	$f(x) \times \frac{1}{f(x)} = 1$
Example	Example
$g(x) = 5x \therefore g^{-1}(x) = \frac{x}{5}$	$g(x) = 5x \therefore \frac{1}{g(x)} = \frac{1}{5x}$

Example 1: An example of the inverse of $y = mx + c$.

Given $f(x) = 2x - 3$, draw $f(x)$ and $f^{-1}(x)$ on the same system of axes.

Given : $f(x) = 2x - 3 \quad \therefore \quad y = 2x - 3$

Step 1: Interchange *x* and *y*: $x = 2y - 3$

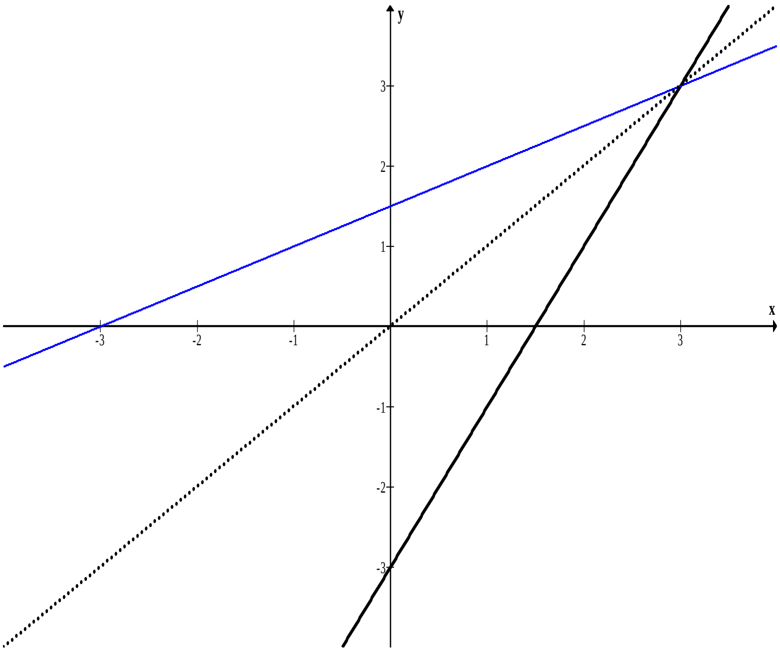
$$x + 3 = 2y$$

$$\frac{x}{2} + \frac{3}{2} = y$$

$$\therefore \quad y = \frac{x}{2} + \frac{3}{2}$$

Therefore, $f^{-1}(x) = \frac{x}{2} + \frac{3}{2}$

Step 2: Sketch the graphs on the same system of axes



The graph of $f^{-1}(x)$ is the reflection of $f(x)$ about the line $y = x$

Please note that when we are dealing with the inverse of a parabola (quadratic function), we encounter the problem that the inverse is not always a function. This is because the quadratic function is not a one-to-one relation (mapping). In order to ensure that we obtain a function for the inverse of the parabola, we must restrict the domain of the original function.

See the example below:

Example 2: An example of the inverse of $y = ax^2$.

Given $f(x) = 3x^2$, draw $f(x)$ and $f^{-1}(x)$ on the same system of axes.

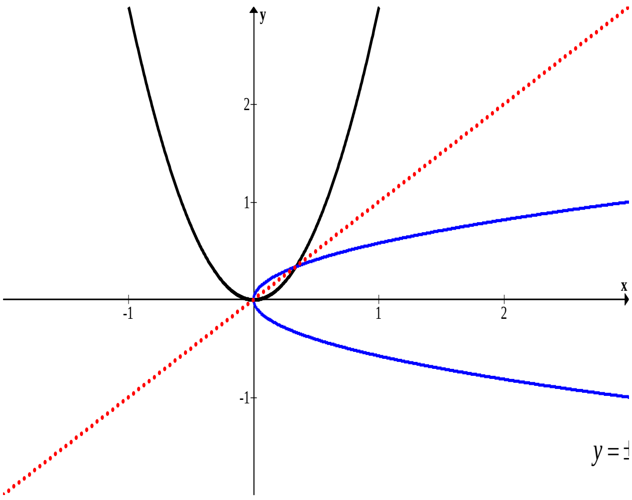
Given : $f(x) = 3x^2 \quad \therefore \quad y = 3x^2$

Step 1: Interchange *x* and *y*: $x = 3y^2$

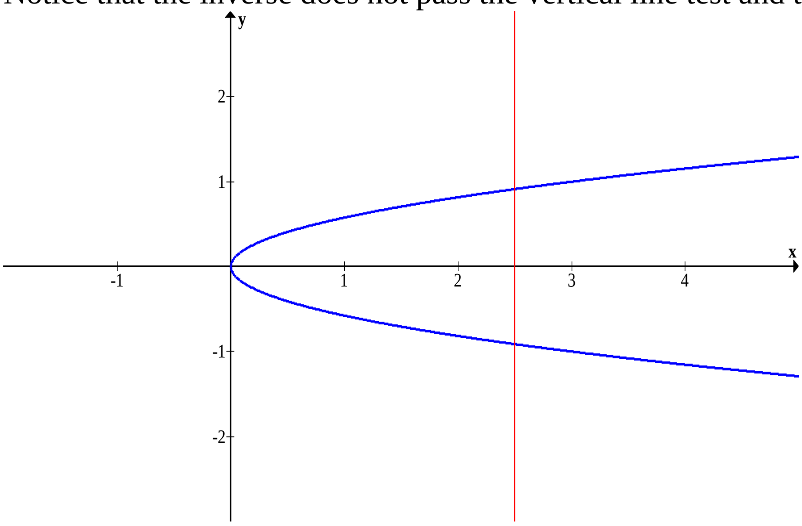
$$\frac{x}{3} = y^2$$

$$\therefore \quad y = \pm\sqrt{\frac{x}{3}} \quad \text{where} \quad x > 0$$

Step 2: Sketch the graphs on the same system of axes



Notice that the inverse does not pass the vertical line test and therefore is not a function.



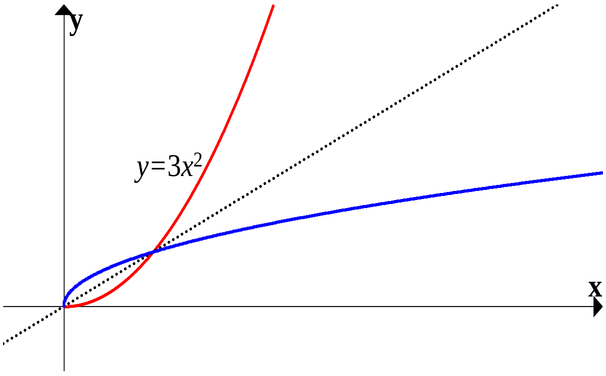
To determine the inverse functions of $y = ax^2$:

1) Interchange x and y: $x = ay^2$
2) Make y the subject of the equation: $\frac{x}{a} = y^2$

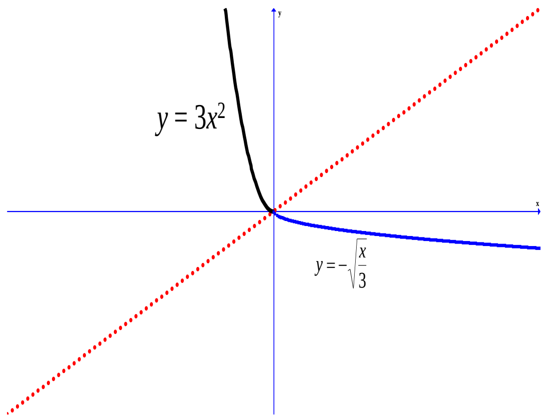
$y = \pm\sqrt{\frac{x}{a}}$ where $x \geq 0$

The vertical line test shows that the inverse of a parabola is not a function. However, we can limit the domain of the parabola so that the inverse of the parabola is a function. We can do this in two ways as illustrated below:

In the this sketch we have restricted the domain to $x \geq 0$.



In the sketch below we have restricted the domain to $x \leq 0$ then $f^{-1}(x) = -\sqrt{\frac{x}{3}}$ would also be a function.



Exercises:

The function concept

1. State if the following are true or false. Provide a reason for each answer.

1.1	The inverse of f = {(2; 3); (4; 7)} is {(3; 2); (7; 4)}	(2)
1.2	f = {(2; -3); (4; 6); (-2; -3); (6; 4)} is a many-to-one relation	(2)
1.3	The inverse of 1.2 is a function	(2)
1.4	The domain of 1.2 is D = {2; 4; 6}	(2)
1.5	The function f and its inverse f^{-1} are reflections in the line $y = -x$	(2)

The inverse of $y = mx + c$

2. Given $f(x) = 2x - 7$

2.1 Is $f(x)$ a function? Explain your answer. (2)

2.2 Write down the domain and range of $f(x)$ (2)

2.3 Determine $f^{-1}(x)$ (2)

2.4 Draw graphs of $f(x)$ and $f^{-1}(x)$ on the same system of axes. (4)

2.5 Give the equation of the line of reflection between the two graphs and indicate this line on the graph using a broken line. (2)

3. Given that $f^{-1}(x) = -2x + 4$, determine $f(x)$. (2)

4. $f(x) = \frac{2}{3}x$ and $g(x) = -3x - 9$. Determine the point(s) of intersection of f^{-1} and g^{-1}. (7)

The inverse of $y = ax^2$

5. Given the function $f(x) = x^2$

5.1 Determine $f^{-1}(x)$. (3)

5.2 Draw the graph of $f^{-1}(x)$. (2)

5.3 Explain why $f^{-1}(x)$ will not be a function? (1)

5.4 Explain how you will restrict the domain of $f(x)$ to ensure that $f^{-1}(x)$ will also be a function. (2)

6. Given $f(x) = \frac{1}{2}x^2$

6.1 Determine the inverse of $f(x)$ (3)

6.2 Is the inverse of $f(x)$ a function or not? Give a reason for your answer. (2)

6.3 How will you restrict the domain of the original function so as to ensure that $f^{-1}(x)$ will also be a function. (1)

6.4 Draw graphs of $f(x)$ and $f^{-1}(x)$ on the same system of axes. (3)

6.5 Determine the point(s) where $f(x)$ and $f^{-1}(x)$ will intersect each other. (4)

7. Given $f(x) = -2x^2$

7.1 Explain why, if the domain of this function is not restricted, its inverse will not be a function? (2)

7.2 Write down the equation of the inverse, $f^{-1}(x)$ of $f(x) = -2x^2$ for $x \in (-\infty\ ;\ 0]$ in the form $f^{-1}(x) = \ldots\ldots..$ (3)

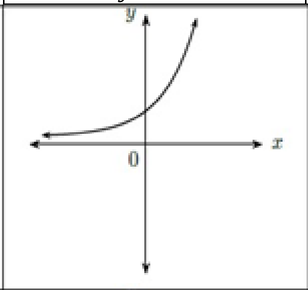
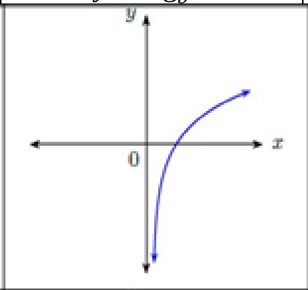
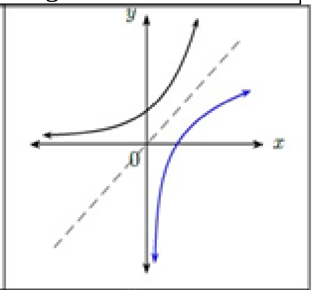
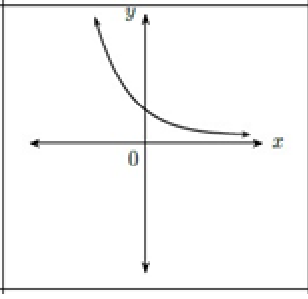
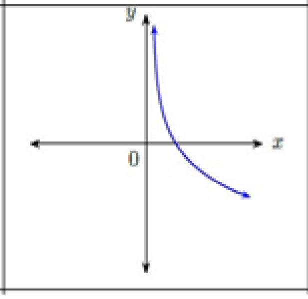
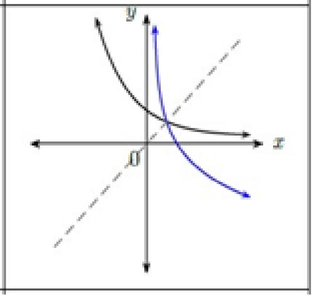
7.3 Write down the domain of $f^{-1}(x)$. (2)

7.4 Draw graphs of both $f(x) = -2x^2$ for $x \in (-\infty\ ;\ 0]$ and $f^{-1}(x)$ on the same system of axes. (4)

Session 2

The Log-function and its inverse:

Summary of graphs: $y = b^x$ **and** $y = \log_b x$

	Exponential function $y = b^x$	Logarithmic function $y = \log_b x$	Exponential & Logarithmic function
$b > 1$			
$0 < b < 1$			

Log and exponential functions as inverses of each other

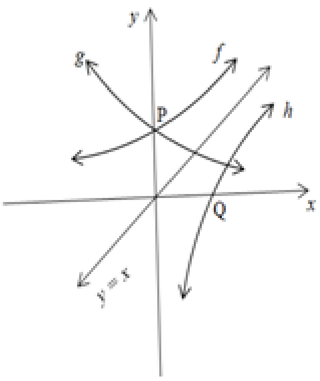
1. The graph alongside shows the functions *g*, *f* and *h*. *f* and *g* are symmetrical with respect to the *y*-axis. *f* and *h* are symmetrical with respect to the line $y = x$.

   If $f(x) = a^x$ and the point (1; 4) lies on $f(x)$:

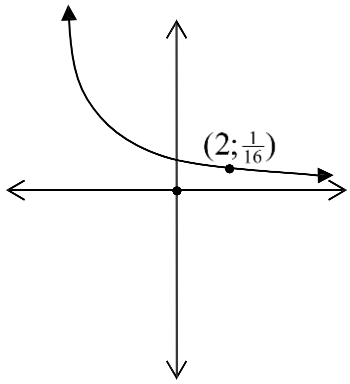
   1.1. Determine the value the value *a*. (2)

   1.2. Write down the coordinates of P and Q. (2)

   1.3. Write down the equation of *g*, *h*. and g^{-1}. (6)



2. The figure represents the graph of $f(x) = a^x$.



2.1 Calculate the value of a. (2)

2.2 Draw a graph of $k(x)$ if k is the inverse of f. Show the intercepts with the axes, as well as the coordinates one other point. Also indicate the asymptotes. (4)

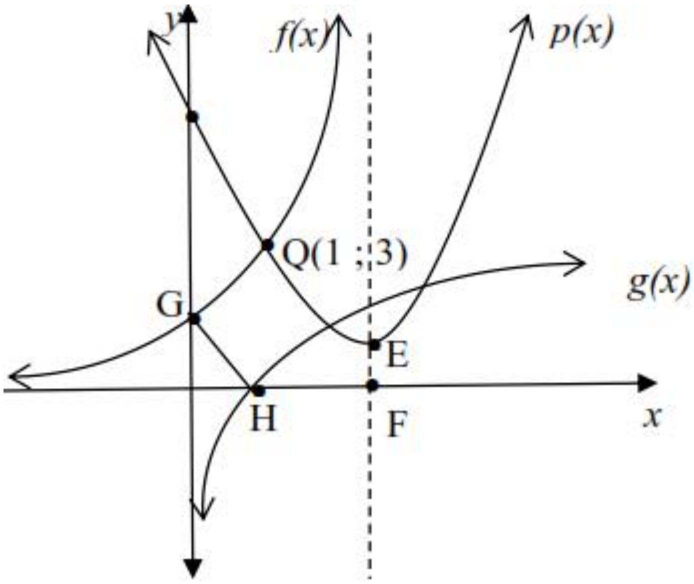
3. The diagram alongside show the functions:

$f(x) = k^x$;

$p(x) = ax^2 + bx + c$ and

$g(x) = \log_m x$

The minimum value of the function $p(x)$ is equal to 1 where $x = 3$. The turning point of the parabola is at F. EF is parallel to the y-axis.



3.1 Determine the values of a, b, c, m and k. (5)

3.2 Calculate the length of EF and GH correct to two decimal places. (3)

3.3 Determine the equation of $f^{-1}(x)$ and $g^{-1}(x)$. (4)

3.4 Hence explain why or why not, $f(x)$ and $g(x)$ will be symmetrical with respect to the line $y = x$. (2)

Session 3: TRIGONOMETRY(± 40/150 Marks)

Compound and Double Angles

In order to master this section it is best to learn the formulae given below. These formulae will also be given on the formulae sheet in the Examination paper.

- Compound Angle Identities:

  (a) $\sin(A-B) = \sin A\cos B - \sin B\cos A$

  $\sin(A+B) = \sin A\cos B + \sin B\cos A$

  (b) $\cos(A-B) = \cos A\cos B + \sin A\sin B$

  $\cos(A+B) = \cos A\cos B - \sin A\sin B$

- Double Angle Identities

  (c) $\sin 2A = 2\sin A\cos A$

  (d) $\cos 2A = \cos^2 A - \sin^2 A$

  $= 1 - 2\sin^2 A$

  $= 2\cos^2 A - 1$

Referred to as double angle formulae

When two angles are added or subtracted to form a new angle, then a compound or a double angle is formed.

What should you ensure you can do at the end of this section for examination purposes:

A. Accepting the Compound Angle formulae $\boxed{\cos(A-B) = \cos A\cos B + \sin A\sin B}$ use it to derive the formulae:

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$
$$\sin(A-B) = \sin A\cos B - \sin B\cos A$$
$$\sin(A+B) = \sin A\cos B + \sin B\cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$\cos 2A = 1 - 2\sin^2 A$$
$$\cos 2A = 2\cos^2 A - 1$$
$$\sin 2A = 2\sin A\cos A$$

*You **must** remember*

Co-functions or Co-ratios

$\sin(90° - \theta) = \cos\theta$

$\cos(90° - \theta) = \sin\theta$

Negative Angles

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = +\cos\theta$

$\tan(-\theta) = -\tan\theta$

$\sin^2\theta + \cos^2\theta = 1$

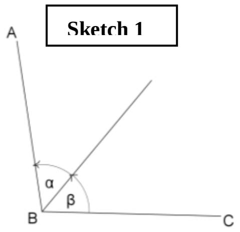
$\sin^2\theta = 1 - \cos^2\theta$

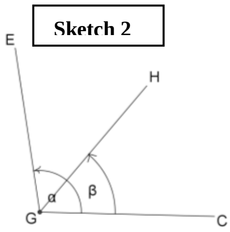
$\cos^2\theta = 1 - \sin^2\theta$

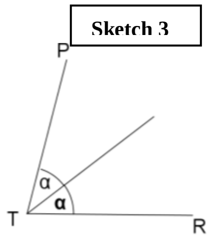
B. Use compound angle and double angle formulae to:

1. Evaluate an expression without using a calculator
2. Simplifying trigonometric expressions
3. Prove identities
4. Solve trigonometric equations (both specific and general solutions)

The sketches below gives a visual of compound and double angles.







Sketch 1: The compound angle $A\hat{B}C$ is equal to the sum of α and β. eg. $75° = 45° + 30°$

Sketch 2: The compound angle $E\hat{G}H$ is equal to the difference between α and β. eg. $15° = 60° - 45°$ or $15° = 45° - 30°$

Sketch 3: The double angle $P\hat{T}R$ is equal to the sum of α and α. eg. $45° = 22.5° + 22.5°$

Given any special angles α and β, we can find the values of the sine and cosine ratios of the angles $\alpha + \beta$, $\alpha - \beta$ and 2α.

Please note:

$0°$; $30°$; $45°$; $60°$ and $90°$ are angles special, because you are able to evaluate any trig function of these angles without using a calculator.

Exercises: Do not use a calculator.

A. Derive each of the compound and double angle formulae in the box on the previous page.

B. 1.

1.1 Evaluate each of the following without using a calculator.

a) $\sin 75°$ b) $\cos 15°$ c) $\cos 105°$ d) $\sin 165°$

e) $\sin 36°.\cos 54° + \cos 36° \sin 54°$ f) $\cos 42°.\cos 18° - \sin 42° \sin 18°$

g) $\sin 85°.\sin 25° + \cos 85° \cos 25°$ h) $\sin 70°.\cos 40° - \cos 70° \sin 40°$

i) $2\sin 30°.\cos 30°$ j) $\dfrac{2\sin 40°.\cos 40°}{\cos 10°}$

1.2 If $\sin\alpha = \dfrac{2}{3}$, $\tan\beta = \sqrt{2}$ and α and β are acute angles determine the value of $\sin(\alpha + \beta)$.

1.3 If $\tan A = \dfrac{2}{3}$ and $90° < A < 360°$, determine without using a calculator $\cos 2A$.

2. Simplify the following expression to a single trigonometric function:

$$\frac{4\cos(-x).\cos(90° + x)}{\sin(30° - x).\cos x + \cos(30° - x).\sin x}$$

3. Prove that

a) $\cos 75° = \dfrac{\sqrt{2}(\sqrt{3} - 1)}{4}$

b) $\cos(90° - 2x).\tan(180° + x) + \sin^2(360° - x) = 3\sin^2 x$

c) $(\tan x - 1)(\sin 2x - 2\cos^2 x) = 2(1 - 2\sin x \cos x)$

4. Determine the general solution for *x* in the following:

a) $\sin 2x.\cos 10° - \cos 2x.\sin 10° = \cos 3x$

b) $\cos^2 x = 3\sin 2x$

c) $2\sin x = \sin(x + 30°)$

ADDITIONAL QUESTIONS

1. Given $\sin\alpha = \dfrac{8}{17}$; where $90^0 \leq \alpha \leq 270°$
With the aid of a sketch and without the use of a calculator, calculate: (3+2+3)
a) $\tan\alpha$ b) $\sin(90° + \alpha)$ c) $\cos 2\alpha$

2. a) Using the expansions for $\sin(A+B)$ and $\cos(A+B)$, prove the identity of:
$$\frac{\sin(A+B)}{\cos(A+B)} = \frac{\tan A + \tan B}{1 - \tan A.\tan B}$$ (3)

b) If $\tan(A+B) = \dfrac{\sin(A+B)}{\cos(A+B)}$, prove in any ΔABC that (4)
$\tan A.\tan B.\tan C = \tan A + \tan B + \tan C$

3. If $\sin 36°\cos 12° = p$ and $\cos 36°\sin 12° = q$, determine in terms of p and q:
a) $\sin 48°$ (3)
b) $\sin 24°$ (3)
c) $\cos 24°$ (3)

4. Show that $\sin^2 20° + \sin^2 40° + \sin^2 80° = \dfrac{3}{2}$
(**HINT:** $40° = 60° - 20°$ **and** $80° = 60° + 20°$ (7)

5. Given: $f(x) = 1 + \sin x$ and $g(x) = \cos 2x$
Calculate the points of intersection of the graphs f and g for $x \in [180°\,;360°]$ (7)

6. Given that $\sin\theta = \dfrac{1}{3}$, calculate the numerical value of $\sin 3\theta$, **WITHOUT** using a calculator. (5)

7. Prove that, for any angle A:
$$\frac{4\sin A\cos A\cos 2A\sin 15°}{\sin 2A(\tan 225° - 2\sin^2 A)} = \frac{\sqrt{6} - \sqrt{2}}{2}$$ (6)

8. Solve for x if $2\cos x = \tan 2x$ and $x \in [-90° \;;\; 90°]$. Show ALL working details. (8)

9. If $\cos\beta = \dfrac{p}{\sqrt{5}}$; where $p < 0$ and $\beta \in [0^0;\; 90^0]$, determine, using a diagram, an expression in terms of p for:
a) $\tan\beta$ (4)
b) $\cos 2\beta$ (3)

10.1 If sin 28° = a and cos 32°= b, determine the following in terms of a and/or b:
a) $\cos 28°$ b) $\cos 64°$ c) $\sin 4°$ (2+3+4)

10.2 Prove without the use of a calculator, that if sin 28° = a and cos 32° = b, then
$b\sqrt{1-a^2} - a\sqrt{1-b^2} = \frac{1}{2}$. (4)

Session 4: Grade 12 Calculus

Cubic Graphs. In this lesson you will work through 3 types of questions regarding graphs

1. Drawing cubic graphs

2. Given the graphs, answer interpretive questions

3. Given the graphs of the derivative, answer interpretive questions

1.1

Given: $f(x) = 2x^3 - x^2 - 4x + 3$

1.1.1 Show that $(x-1)$ is a factor of $f(x)$. (2)

1.1.2 Hence factorise $f(x)$ completely. (2)

1.1.3 Determine the co-ordinates of the turning points of f. (4)

1.1.4 Draw a neat sketch graph of f indicating the co-ordinates of the turning points as well as the x-intercepts. (4)

1.1.5 For which value of x will f have a point of inflection? (4)

[16]

1.2

Given $f(x) = x^3 + x^2 - 5x + 3$

1.2.1 Show that $(x-1)$ is a factor of $f(x)$. (2)

1.2.2 Factorise $f(x)$ fully. (3)

1.2.3 Determine the x and y intercepts of $f(x)$. (2)

1.2.4 Determine the co-ordinates of the turning point(s) of $f(x)$. (4)

1.2.5 Find the x-coordinate of the point of inflection of $f(x)$. (1)

1.2.6 Draw a sketch graph of $f(x)$. (2)

1.2.7 For which value(s) of x is $f(x)$ increasing? (2)

1.2.8 Describe one transformation of $f(x)$ that, when applied, will result in $f(x)$ having two unequal positive real roots. (2)

1.2.9 Give the equation of g if g is the reflection of f in the y-axis. (3)

1.2.10 Determine the average rate of change of f between the points (0 ; 3) and (1;0). (2)

1.2.11 Determine the equation of the tangent to the f when x = -2. (4)

1.2.12 Prove that the tangent in 1.2.11 will intersect or touch the curve of f at two places. (4)

1.4

A cubic function f has the following properties:

- $f\left(\frac{1}{2}\right) = f(3) = f(-1) = 0$
- $f'(2) = f'\left(-\frac{1}{3}\right) = 0$

f decreases for $x \in \left[-\frac{1}{3}\,;\,2\right]$ only

Draw a possible sketch graph of f, clearly indicating the x-coordinates of the turning points and ALL the x-intercepts. (4)

1.3

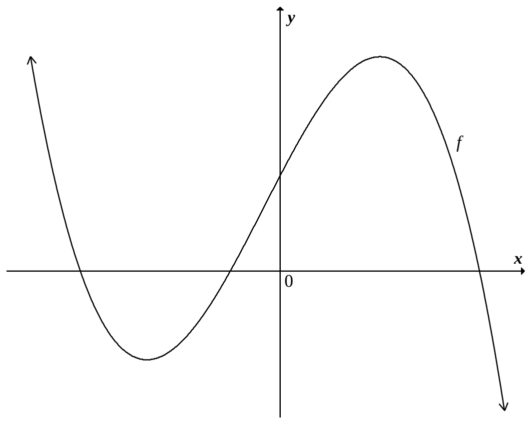
The tangent to the curve of $g(x) = 2x^3 + px^2 + qx - 7$ at $x = 1$ has the equation $y = 5x - 8$.

1.3.1 Show that (1 ; – 3) is the point of contact of the tangent to the graph. (1)

1.3.2 Hence or otherwise, calculate the values of p and q. (6)

2.1

The graph of the function $f(x) = -x^3 - x^2 + 16x + 16$ is sketched below.

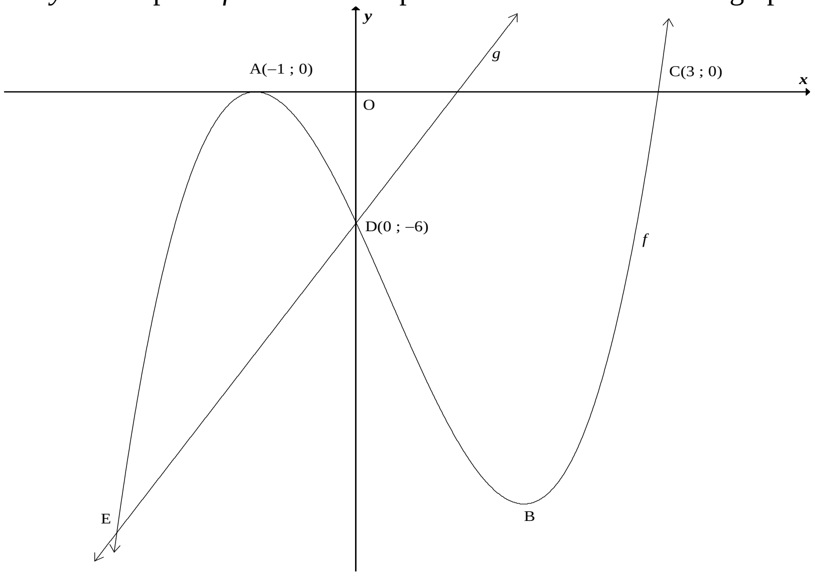


2.1.1 Calculate the x-coordinates of the turning points of f. (4)

2.1.2 Calculate the x-coordinate of the point at which $f'(x)$ is a maximum. (3)

2.2

The graphs of $f(x)=ax^3+bx^2+cx+d$ and $g(x)=6x-6$ are sketched below. A(– 1 ; 0) and C(3 ; 0) are the x-intercepts of f. .The graph of f has turning points at A and B. D(0 ; – 6) is the y-intercept of f. E and D are points of intersection of the graphs of f and g.



2.2.1 Show that $a = 2$; $b = – 2$; $c = – 10$ and $d = – 6$. (5)

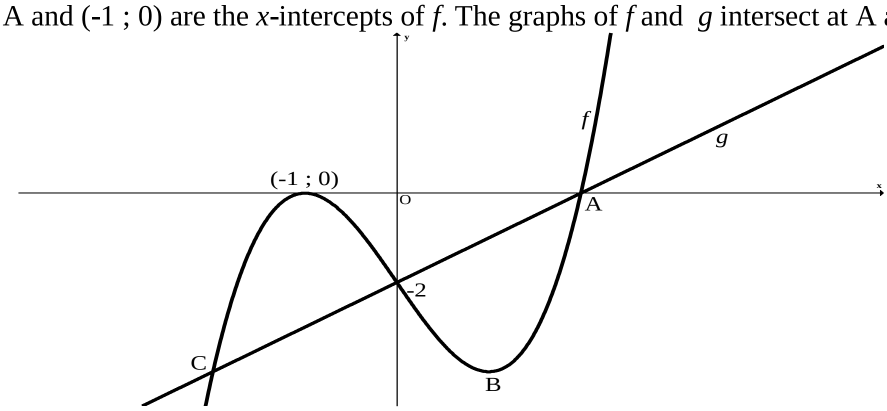
2.2.2 Calculate the coordinates of the turning point B. (5)

2.2.3 $h(x)$ is the vertical distance between $f(x)$ and $g(x)$, that is $h(x)= f(x)-g(x)$. (5)

Calculate x such that $h(x)$ is a maximum, where $x < 0$.. [15]

2.3

The graph below represents the function f and g with $f(x)=ax^3-cx-2$ and $g(x)=x-2$.
A and (-1 ; 0) are the x-intercepts of f. The graphs of f and g intersect at A and C.



2.3.1 Determine the coordinates of A. (1)

2.3.2 Show by calculations that $a = 1$ and $c = -1$. (4)

2.3.3 Determine the coordinates of B, a turning point of f. (3)

2.3.4 Show that the line BC is parallel to the x-axis. (7)

2.3.5 Find the x-coordinate of the point of inflection of f. (2)

2.3.6 Write down the values of k for which $f(x)=k$ will have only ONE root. (3)

2.3.7 Write down the values of x for which $f'(x)<0$. (2)

2.4 Consider the graph of $g(x) = -2x^2 - 9x + 5$.

2.5.1 Determine the equation of the tangent to the graph of g at $x = -1$. (4)

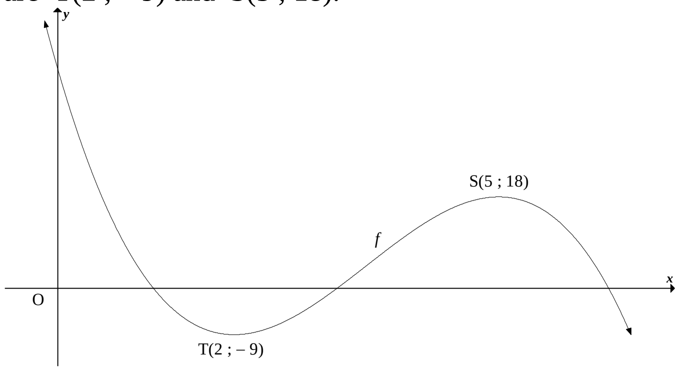
2.5.2 For which values of q will the line $y = -5x + q$ not intersect the parabola? (3)

2.5 Given: $h(x) = 4x^3 + 5x$

Explain if it is possible to draw a tangent to the graph of h that has a negative gradient. Show ALL your calculations. (3)

2.6

The function $f(x) = -2x^3 + ax^2 + bx + c$ is sketched below. The turning points of the graph of f are T(2 ; – 9) and S(5 ; 18).



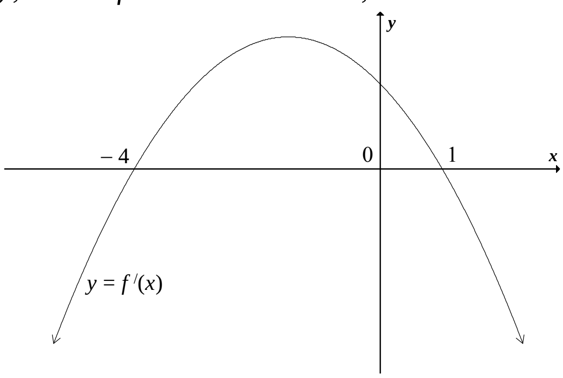
2.6.1 Show that $a = 21$, $b = -60$ and $c = 43$. (7)

2.6.2 Determine an equation of the tangent to the graph of f at $x = 1$. (5)

2.6.3 Determine the x-value at which the graph of f has a point of inflection. (2)

3.1

The graph of $y = f'(x)$, where f is a cubic function, is sketched below.



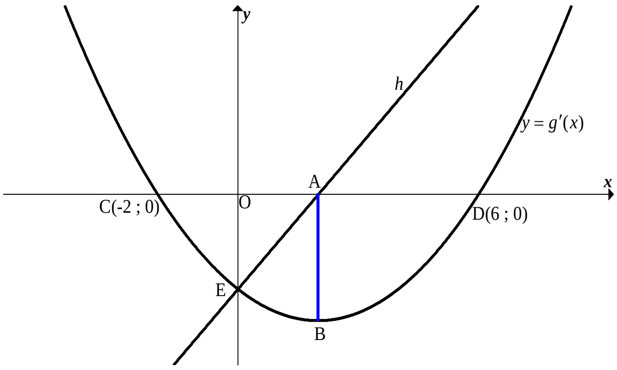
Use the graph to answer the following questions:

3.1.1 For which values of x is the graph of $y = f'(x)$ decreasing? (1)

3.1.2 At which value of x does the graph of f have a local minimum? Give reasons for your answer. (3)

3.2

The graphs of $y = g'(x) = ax^2 + bx + c$ and $h(x) = 2x - 4$ are sketched below. The graph of $y = g'(x) = ax^2 + bx + c$ is a derivative graph of a cubic function g. The graphs of h and g' have a common y-intercept at E. C(-2 ; 0) and D(6 ; 0) are the x-intercept of the graph of g'. A is the x-intercept of h and B is the turning point of g'. AB $\parallel$ y-axis.



3.2.1 Write down the coordinates of E. (1)

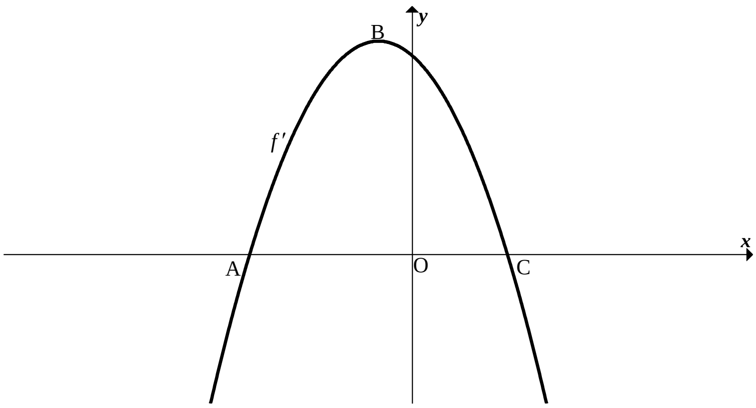
3.2.2 Determine the equation of the graph of g' in the form $y = ax^2 + bx + c$. (4)

3.2.3 Write down the x-coordinates of the turning points of g. (2)

3.2.4 Write down the x-coordinates of the point of inflection of the graph of g. (2)

3.2.5 Explain why g has a local maximum at $x = -2$ (3)

3.3 Sketched below is the graph of f', the derivative of $f(x) = -2x^3 - 3x^2 + 12x + 20$. A, B and C are the intercepts of f' with the axes.



3.3.1 Determine the coordinates of A. (2)

3.2.2 Determine the coordinates of B and C. (3)

3.2.3 Which points on the graph of $f(x)$ will have exactly the same x-coordinates as B and C? (1)

3.2.4 For which values of x will $f(x)$ be increasing? (2)

3.2.5 Determine the y-coordinates of the point of inflection of f. (4)

Session 5: Grade 11 geometry

Below are Grade 11 Theorems, Converse Theorems and their Corollaries which you must know. The proofs of the theorems marked with (**) must be studied because it could be examined.

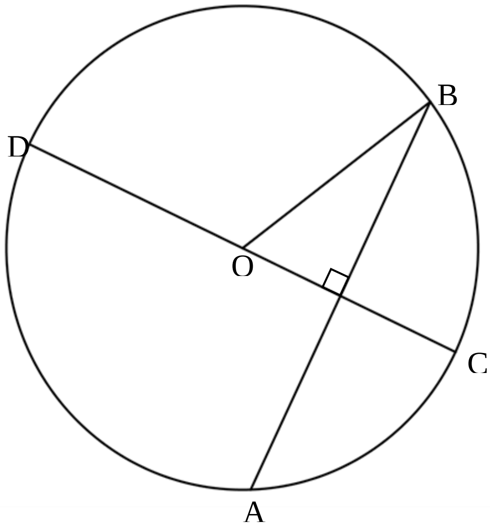
1	**Theorem****	The line drawn from the centre of a circle perpendicular to a chord bisects the chord; **(line from centre to midpt of chord)**
	Converse	The line from the centre of a circle to the midpoint of a chord is perpendicular to the chord. **(line from centre ⊥ to chord)**
		The perpendicular bisector of a chord passes through the centre of the circle; **(perp bisector of chord)**
2	**Theorem****	The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre); **(∠ at centre = 2 ×∠ at circumference)**
	Corollary	1. Angle in a semi-circle is 90^0 **(∠s in semi circle)** 2. Angles subtended by a chord of the circle, on the same side of the chord, are equal **(∠s in the same seg)** 3. Equal chords subtend equal angles at the circumference **(equal chords; equal ∠s)** 4. Equal chords subtend equal angles at the centre **(equal chords; equal ∠s)** 5. Equal chords in equal circles subtend equal angles at the circumference of the circles. **(equal circles; equal chords; equal ∠s)**
	Corollary Converse	1. If the angle subtended by a chord at the circumference of the circle is 90°, then the chord is a diameter. **(converse ∠s in semi circle)** 2. If a line segment joining two points subtends equal angles at two points on the same side of the line segment, then the four points are concyclic.
3	**Theorem****	The opposite angles of a cyclic quadrilateral are supplementary; **(opp ∠s of cyclic quad)**
	Converse	If the opposite angles of a quadrilateral are supplementary then the quadrilateral is a cyclic quadrilateral. **(opp ∠s quad sup OR converse opp ∠s of cyclic quad)**
	Corollary	The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle of the quadrilateral. **(ext ∠ of cyclic quad)**
	Corollary Converse	If the exterior angle of a quadrilateral is equal to the interior opposite angle of the quadrilateral, then the quadrilateral is cyclic. **(ext ∠ = int opp ∠ OR converse ext ∠ of cyclic quad)**
4	**Theorem**	The tangent to a circle is perpendicular to the radius/diameter of the circle at the point of contact. **(tan ⊥ radius)**
	Converse	If a line is drawn perpendicular to a radius/diameter at the point where the radius/diameter meets the circle, then the line is a tangent to the circle. **(line ⊥ radius)**
5	**Theorem**	Two tangents drawn to a circle from the same point outside the circle are equal in length. **(Tans from common pt OR Tans from same pt)**
6	**Theorem****	The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment. **(tan chord theorem)**
	Converse	If a line is drawn through the end-point of a chord, making with the chord an angle equal to an angle in the alternate segment, then the line is a tangent to the circle. **(converse tan chord theorem OR ∠ between line and chord)**

The theory of quadrilaterals must be revised because it will be integrated into geometry questions in the examination.

Question 1

In the diagram below, O is the centre of the circle. Chord AB is perpendicular to diameter DC.

CM : MD = 4 : 9 and AB = 24 units.



1.1 Determine an expression for DC in terms of x if CM = $4x$ units.

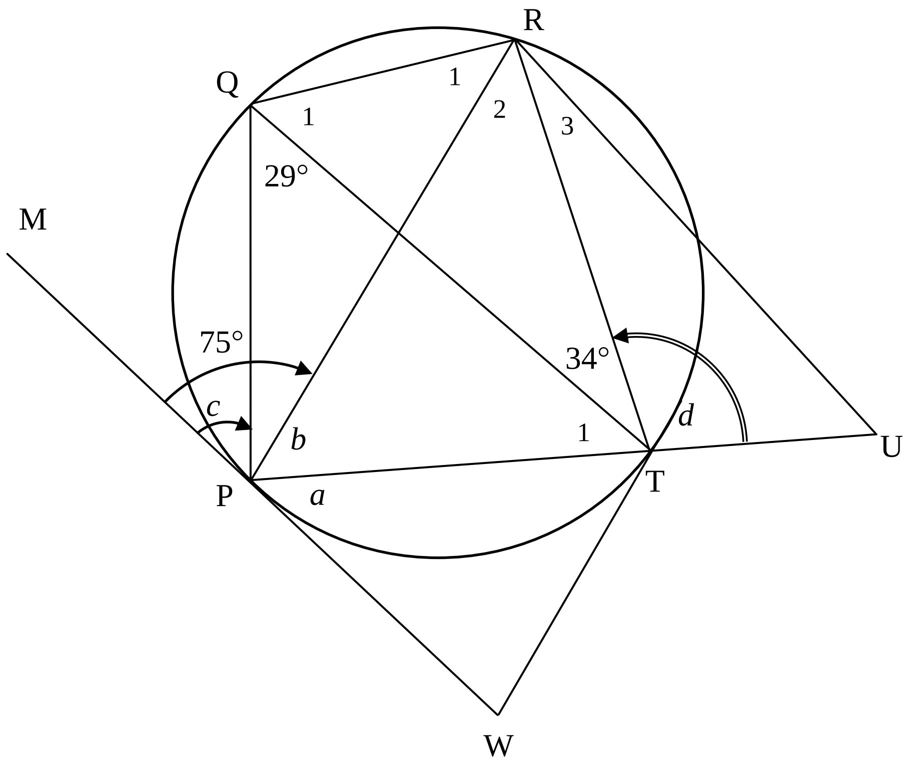
1.2 Determine an expression for OM in terms of x.

1.3 Hence, or otherwise, calculate the length of the radius.

Question 2

In the diagram points P, Q, R and T lie on the circumference of a circle. MW and TW are tangents to the circle at P and T respectively. PT is produced to meet RU at U. $M\hat{P}R = 75°$, $P\hat{Q}T = 29°$ and $Q\hat{T}R = 34°$.

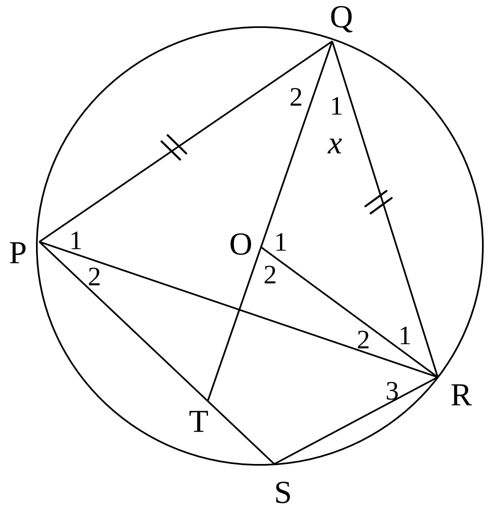
Let $T\hat{P}W = a$, $R\hat{P}T = b$, $M\hat{P}Q = c$ and $R\hat{T}U = d$, calculate the values of *a*, *b*, *c* and *d*.



Question 3

In the diagram below, O is the centre of the circle. P, Q, R and S are points on the circumference of the circle. TOQ is a straight line such that T lies on PS.

PQ = QR and $\hat{Q}_1 = x$.



3.1 Calculate, with reasons, $\hat{P}_1$ in terms of x. (3)

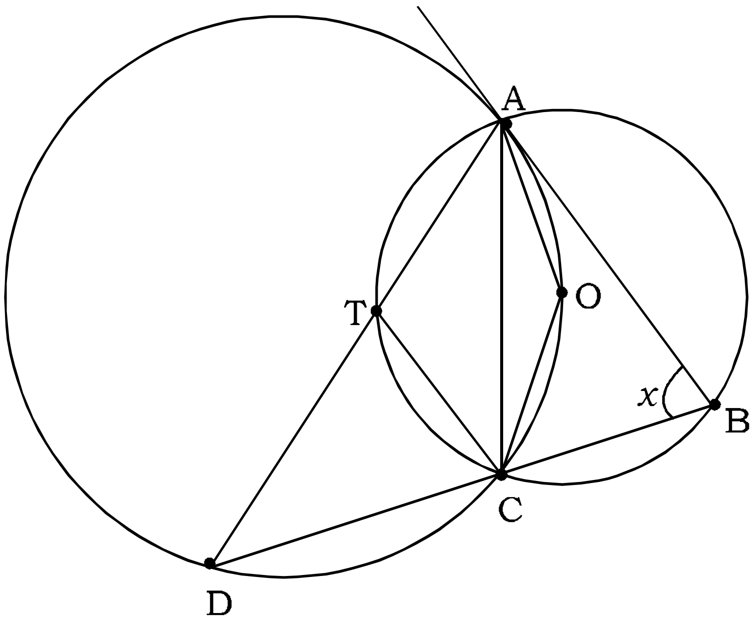
3.2 Show that TQ bisects $P\hat{Q}R$. (3)

3.3 Show that STOR is a cyclic quadrilateral. (3)

Question 4

In the diagram two circles intersect in A and C. BA is a tangent to the larger circle at point A. The straight lines ATD and BCD cut the circles in T and D, as well as C and D respectively. The larger circle passes through the centre O of the smaller circle.

Let $\hat{B} = x$.



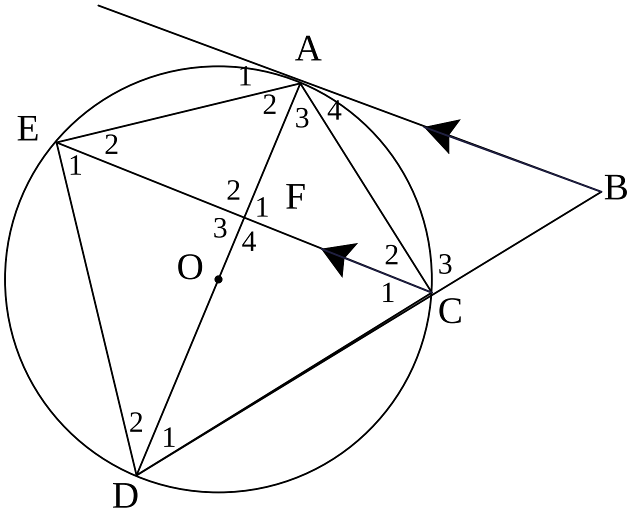
4.1 Prove that $\hat{D} = 180° - 2x$. (4)

4.2 Prove that AD = BD. (5)

4.3 Prove that TC || AB. (2)

Question 5

In the figure below, AB is a tangent to the circle with centre O. AC = AO and BA // CE. DC produced cuts tangent BA at B.



Prove stating reasons, that:

i. $\hat{C}_2 = \hat{D}_1$ (4)

ii. ΔACF /// ΔADC (5)

iii. AD = 4 AF (5)

Question 6

ABCD is a cyclic quadrilateral. AC//BP, PA and PB are tangents. If $B\hat{A}P = x$

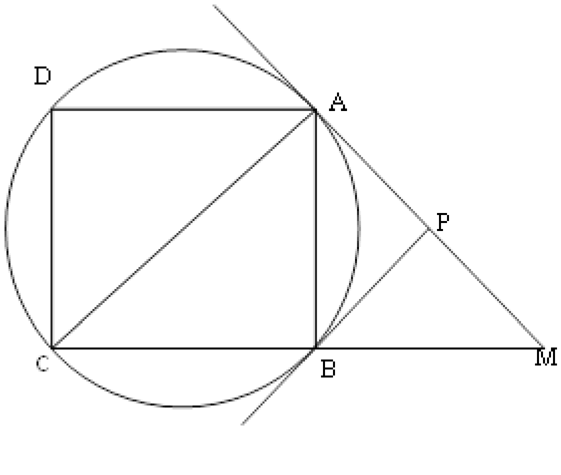
6.1 Name two angles in terms of *x*.

6.2 Prove that ΔABC is isosceles triangle.

6.3 Give $\hat{M}$ in terms of *x*.

6.4 If ABCD is a square prove that:

Δ APB $\equiv$ Δ BPM



Session 6
Grade 12 Geometry

Below are **Grade 12 Theorems**, Converse Theorems and their Corollaries which you must know. The proofs of the theorems marked with (**) must be studied because it could be examined.

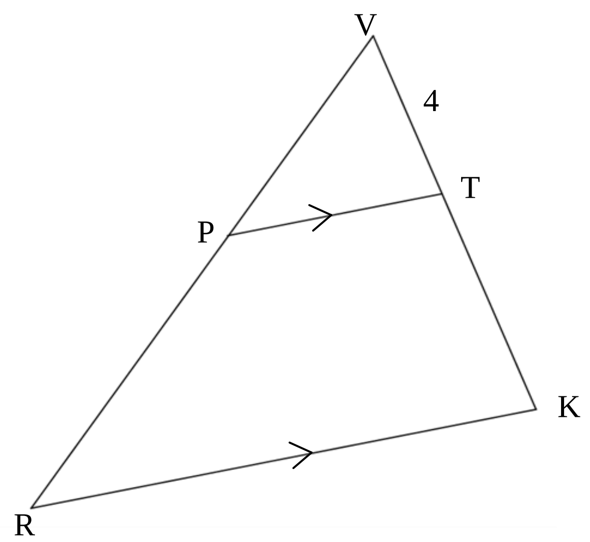
1	**Theorem****	A line drawn parallel to one side of a triangle divides the other two sides proportionally. **(line || one side of Δ OR prop theorem; name || lines)**
	Converse	If a line divides two sides of a triangle in the same proportion, then the line is parallel to the third side. **(line divides two sides of Δ in prop)**
	Theorem**	If two triangles are equiangular, then the corresponding sides are in proportion (and consequently the triangles are similar) **(||| Δs OR equiangular Δs)**
	Converse	If the corresponding sides of two triangles are proportional, then the triangles are equiangular (and consequently the triangles are similar). **(Sides of Δ in prop)**

Question 1

1.1

In the diagram below, ΔVRK has P on VR and T on VK such that PT || RK. VT = 4 units, PR = 9 units, TK = 6 units and VP = $2x - 10$ units.

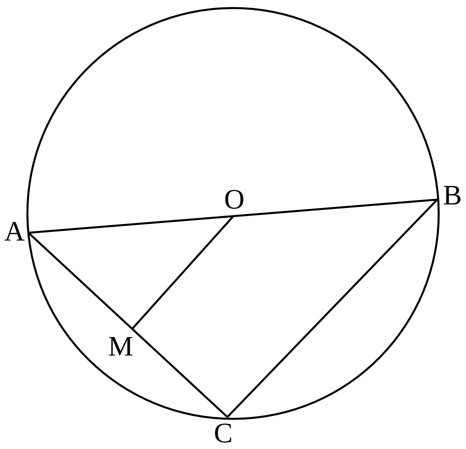
Calculate the value of x.



1.2

O is the centre of the circle below. OM $\perp$ AC. The radius of the circle is equal to 5 cm and

BC = 8 cm.



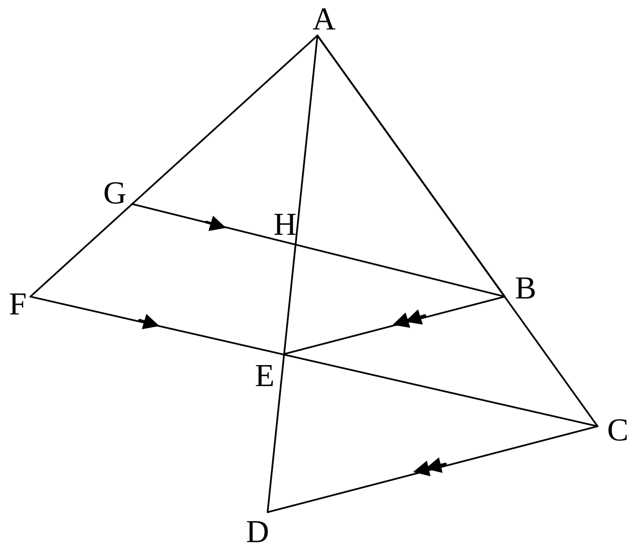
1.2.1 Write down the size of $B\hat{C}A$. (1)

1.2.2 Calculate:

(a) The length of AM, with reasons (3)

(b) Area ΔAOM : Area ΔABC (3)

1.3 In the figure below, GB || FC and BE || CD. AC = 6 cm and $\dfrac{AB}{BC} = 2$.



1.3.1 Calculate with reasons:

AH : ED (4)

$\dfrac{BE}{CD}$ (2)

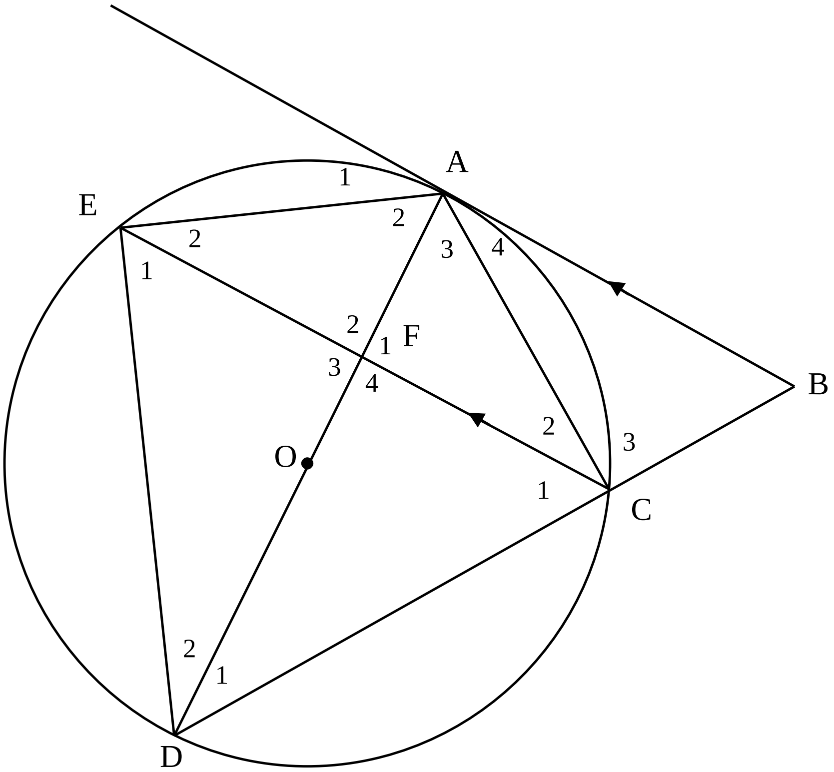
1.3.2 If HE = 2 cm, calculate the value of AD × HE. (2)

[8]

Question 2

2.1

In the figure below, AB is a tangent to the circle with centre O. AC = AO and BA || CE. DC produced, cuts tangent BA at B.



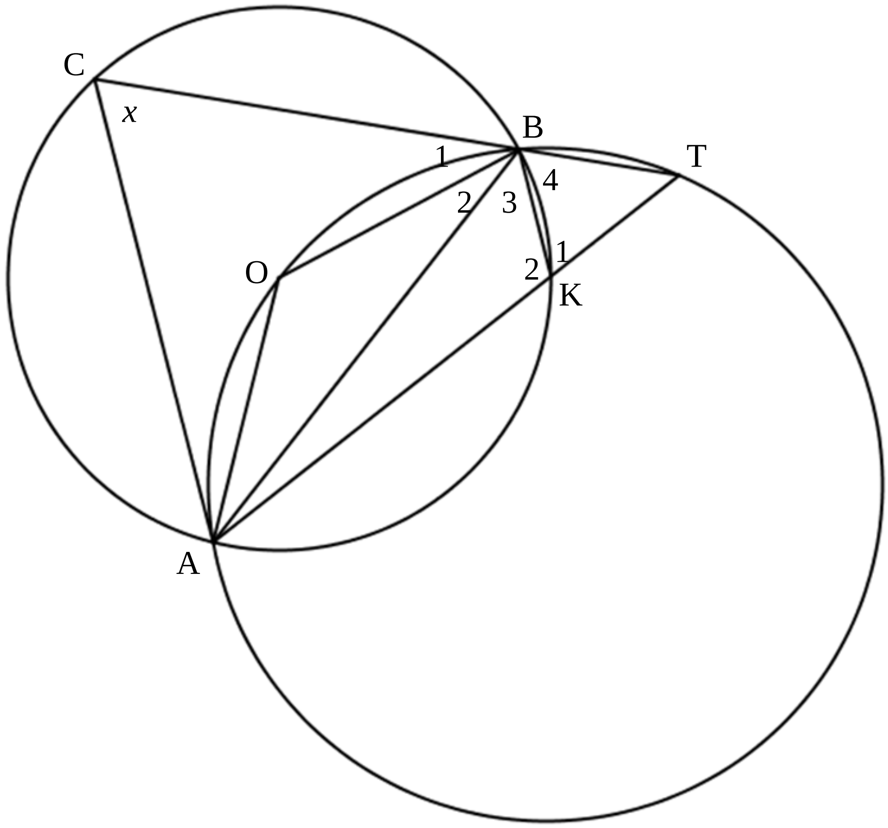
2.1.1 Show $\hat{C}_2 = \hat{D}_1$. (3)

2.1.2 Prove that ΔACF ||| ΔADC. (3)

2.1.3 Prove that AD = 4AF. (4)

2.2

O is the centre of the circle CAKB. AK produced intersects circle AOBT at T. $A\hat{C}B = x$



2.2.1 Prove that $\hat{T} = 180° - 2x$. (3)

2.2.2 Prove that AC || KB. (5)

2.2.3 Prove that ΔBKT ||| ΔCAT (3)

2.2.4 If AK : KT = 5 : 2, determine the value of $\frac{AC}{KB}$ (3)

[14]