basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATION

MATHEMATICS P2

2015

MARKS: 150

TIME: 3 hours

This question paper consists of 14 pages, 1 information sheet and an answer book of 18 pages.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, et cetera which you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Write neatly and legibly.

QUESTION 1

The data below shows the ages (in years) of people who visited the library between 08:00 and 09:00 on a certain morning.

3	4	4	5	23	29	32	36	40	47	56	66	68	76	82

1.1 Determine:

1.1.1 The mean age of the visitors (2)

1.1.2 The median of the data (1)

1.1.3 The interquartile range of the data (3)

1.1.4 The standard deviation of the data (2)

1.2 Draw a box and whisker diagram for the data. (3)

1.3 By making reference to the box and whisker diagram, comment on the skewness of the data set. (1)
[12]

QUESTION 2

Fourteen learners attended a Geometry training course spread over 12 Saturdays. Learners wrote a Geometry test at the end of the course. One learner was absent for this test. The number of Saturdays attended and the mark (as a %) each learner obtained for the test are shown in the table below.

Number of Saturdays attended	12	11	10	10	9	9	7	6	5	4	12	11	6
Mark (as a %)	96	91	78	83	75	62	70	68	56	34	88	90	59

2.1 Calculate the equation of the least squares regression line. (3)

2.2 Calculate the correlation coefficient. (2)

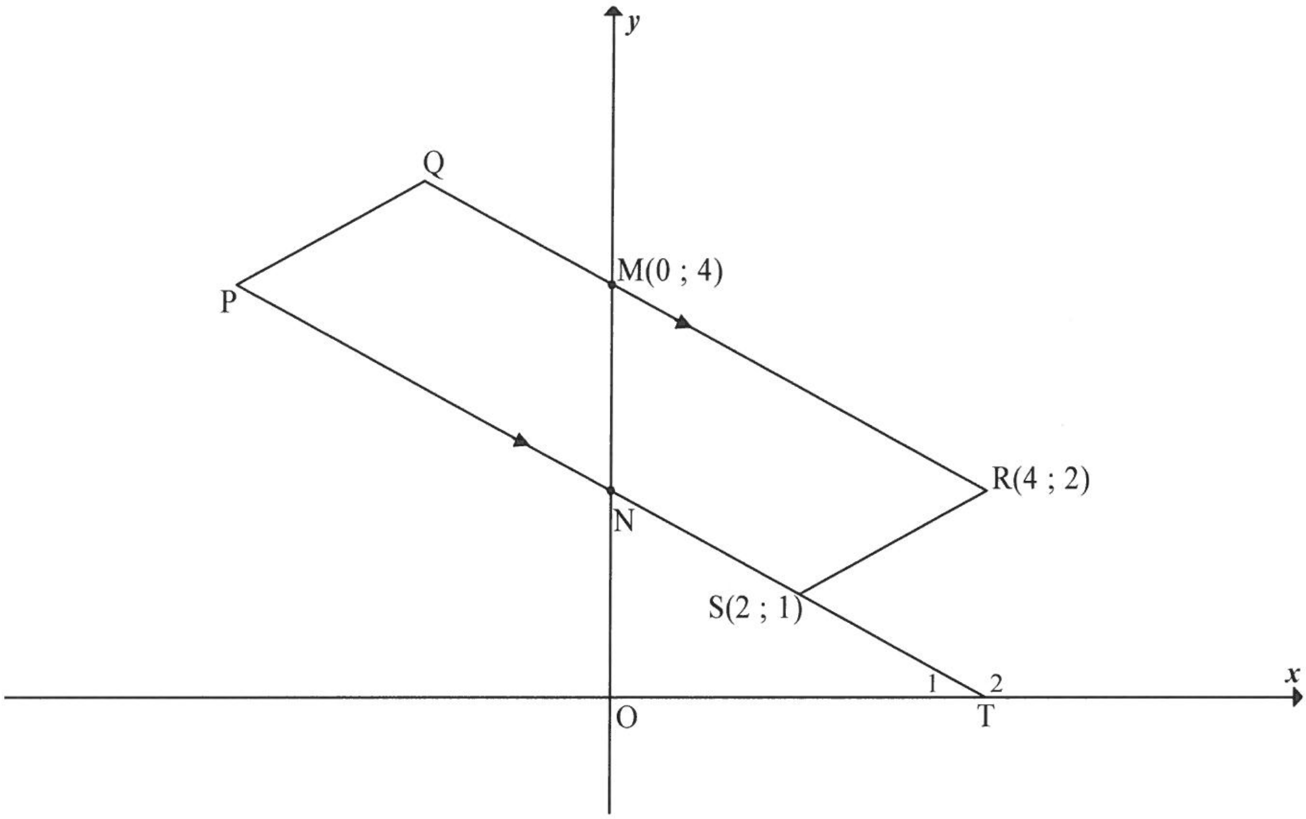
2.3 Comment on the strength of the relationship between the variables. (1)

2.4 The learner who was absent for the test attended the course on 8 Saturdays. Predict the mark that this learner would have scored for the test. (2)

[8]

QUESTION 3

In the diagram below P, Q, R(4 ; 2) and S(2 ; 1) are the vertices of a quadrilateral with PS | | QR. M(0 ; 4) and N are the y-intercepts of QR and PS respectively. PS produced cuts the x-axis at T.



3.1 Calculate the gradient of RS. (2)

3.2 Given that the equation of PQ is $2y = x + 12$, calculate, with reasons, the length of PQ. (4)

3.3 Determine the equation of PT in the form $y = mx + c$. (4)

3.4 Hence, write down the coordinates of N. (1)

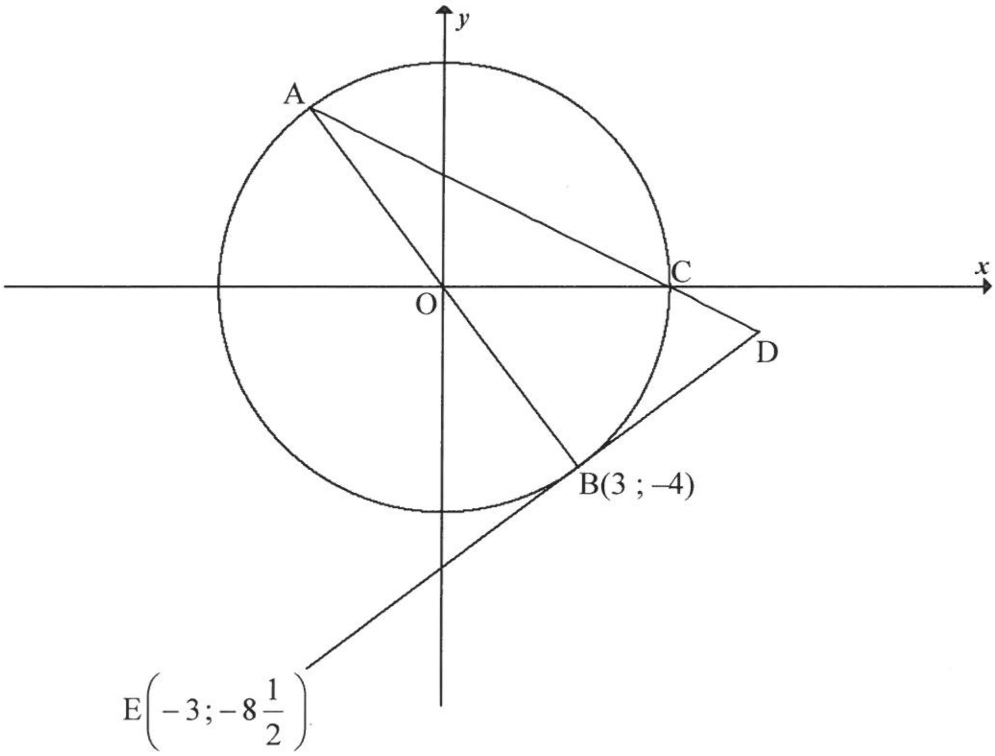
3.5 Calculate, with reasons, the size of $R\hat{N}S$, rounded to ONE decimal place. (5)

[16]

QUESTION 4

The diagram below shows a circle with centre O at the origin. AB is a diameter of the circle. The straight line ACD meets the tangent EBD to the circle at D.

The coordinates of B and E are (3 ; – 4) and $\left(-3;-8\frac{1}{2}\right)$ respectively.



4.1 Determine the coordinates of A. (2)

4.2 Determine the equation of the circle passing through A, B and C. (3)

4.3 Write down the length of AB. (2)

4.4 If it is given that AD is $\sqrt{125}$ units, calculate the length of BD. Give reasons. (3)

4.5 Calculate the area of ΔABD. (3)

4.6 Another circle passes through A, B and E. Determine, with reasons, the equation of this circle. Write the answer in the form $(x-a)^2+(y-b)^2=r^2$. (6)

[19]

QUESTION 5

5.1 Given that $\cos\beta = -\dfrac{1}{\sqrt{5}}$, where $180° < \beta < 360°$.

Determine, with the aid of a sketch and without using a calculator, the value of $\sin\beta$. (5)

5.2 Determine the value of the following expression:

$$\frac{\tan(180° - x).\sin(x - 90°)}{4\sin(360° + x)}$$ (6)

5.3 If $\sin A = p$ and $\cos A = q$:

5.3.1 Write $\tan A$ in terms of p and q (1)

5.3.2 Simplify $p^4 - q^4$ to a single trigonometric ratio (4)

5.4 Consider the identity: $\dfrac{\cos\theta}{\sin\theta} - \dfrac{\cos 2\theta}{\sin\theta.\cos\theta} = \tan\theta$

5.4.1 Prove the identity. (5)

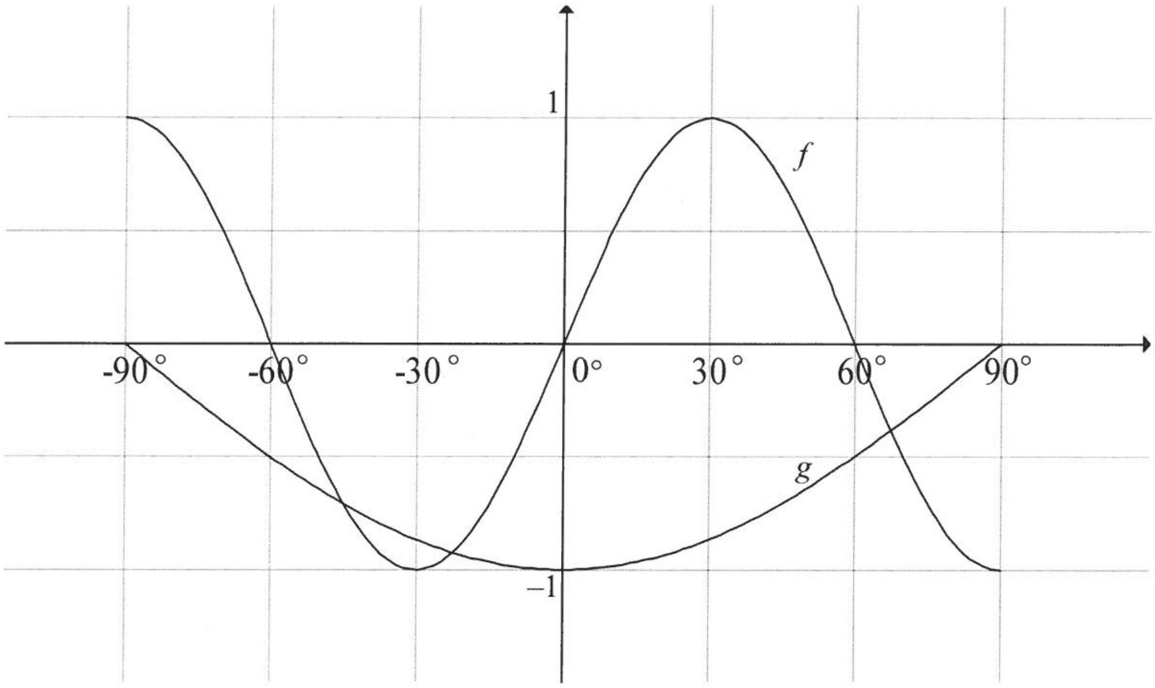
5.4.2 For which value(s) of θ in the interval $0° < \theta < 180°$ will the identity be undefined? (2)

5.5 Determine the general solution of $2\sin 2x + 3\sin x = 0$ (6)

[29]

QUESTION 6

In the diagram below the graphs of $f(x) = \sin bx$ and $g(x) = -\cos x$ are drawn for $-90° \leq x \leq 90°$. Use the diagram to answer the following questions.



6.1 Write down the period of f. (1)

6.2 Determine the value of b. (1)

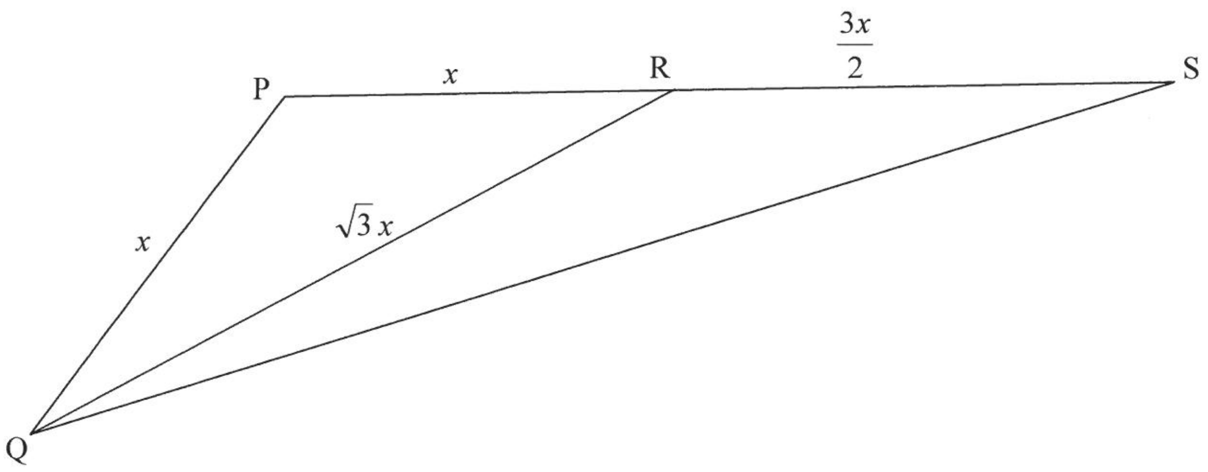
6.3 The general solutions of the equation $\sin bx = -\cos x$ are $x = 67{,}5° + k.90°$ or $x = 135° + k.180°$ where $k \in Z$.
Determine the x-values of the points of intersection of f and g for the given domain. (3)

6.4 Write down the values of x for which $\sin bx + \cos x < 0$ for the given domain. (4)
[9]

QUESTION 7

Triangle PQS forms a certain area of a park. R is a point on PS and QR divides the area of the park into two triangular parts, as shown below, for a festive event.

PQ = PR = x units, RS = $\frac{3x}{2}$ units and RQ = $\sqrt{3}x$ units.



7.1 Calculate the size of $\hat{P}$. (4)

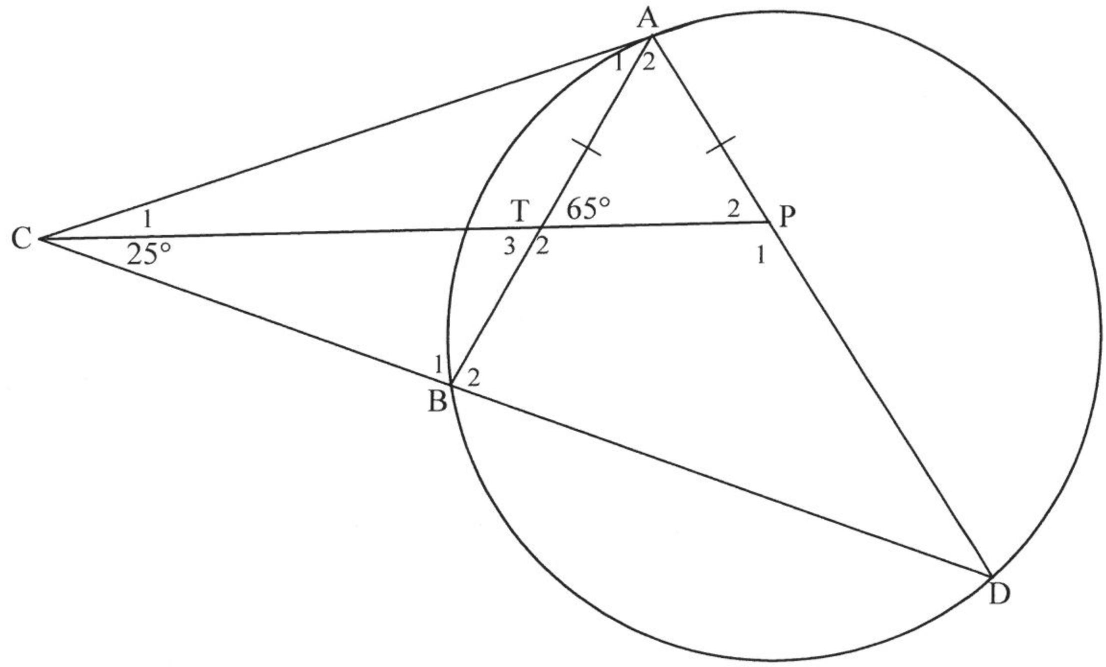
7.2 Hence, calculate the area of triangle QRS in terms of x in its simplest form. (5)

[9]

Give reasons for ALL statements in QUESTIONS 8, 9, 10 and 11.

QUESTION 8

In the diagram ΔACD is drawn with points A and D on the circumference of a circle. CD cuts the circle at B. P is a point on AD with CP the bisector of $A\hat{C}D$. CP cuts the chord AB at T. $AT = AP$, $A\hat{T}P = 65°$ and $P\hat{C}D = 25°$.



8.1 Determine the size of each of the following:

8.1.1 $\hat{P}_2$ (2)

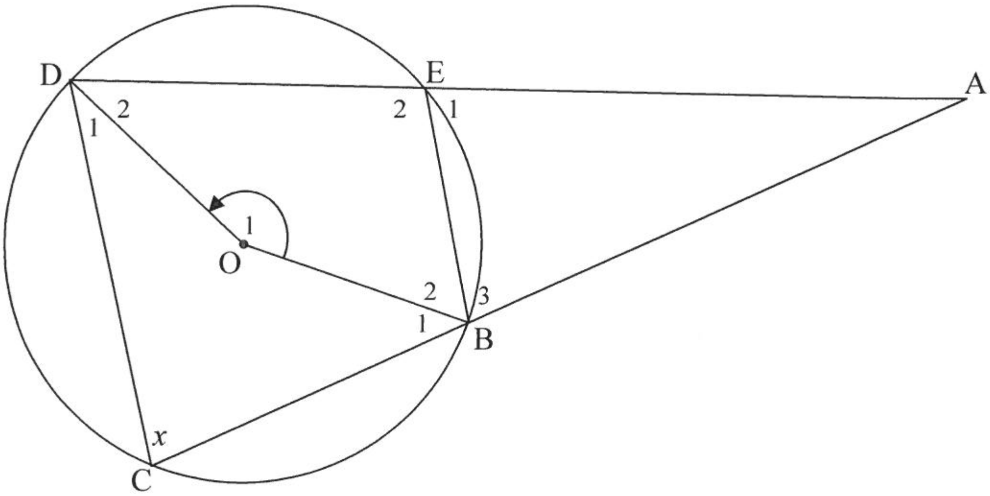
8.1.2 $\hat{D}$ (2)

8.1.3 $\hat{A}_1$ (2)

8.2 Is CA a tangent to the circle ABD? Motivate your answer. (2)
[8]

QUESTION 9

In the diagram O is the centre of the circle and BO and OD are drawn. Chords CB and DE are produced to meet in A. Chords BE and CD are drawn. $B\hat{C}D = x$.



9.1 Give the reason for each of the statements in the table. Complete the table provided in the ANSWER BOOK by writing down the reason for each statement. (2)

	Statement	Reason
9.1.1	$\hat{E}_1 = x$	
9.1.2	$\hat{O}_1 = 2x$	

9.2 If it is given that BE | | CD, prove that:

9.2.1 AC = AD (4)

9.2.2 ABOD is a cyclic quadrilateral (3)

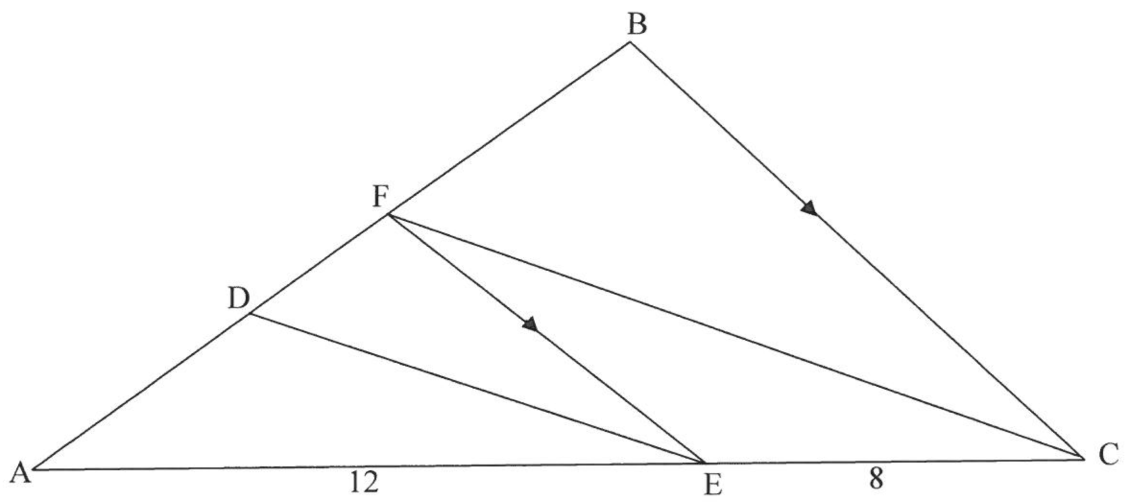
[9]

QUESTION 10

10.1 Complete the following statement of the theorem in the ANSWER BOOK:

If a line divides two sides of a triangle in the same proportion, then ... (1)

10.2 In the diagram ABC is a triangle with F on AB and E on AC. BC | | FE. D is on AF with $\frac{AD}{AF} = \frac{3}{5}$. AE = 12 units and EC = 8 units.



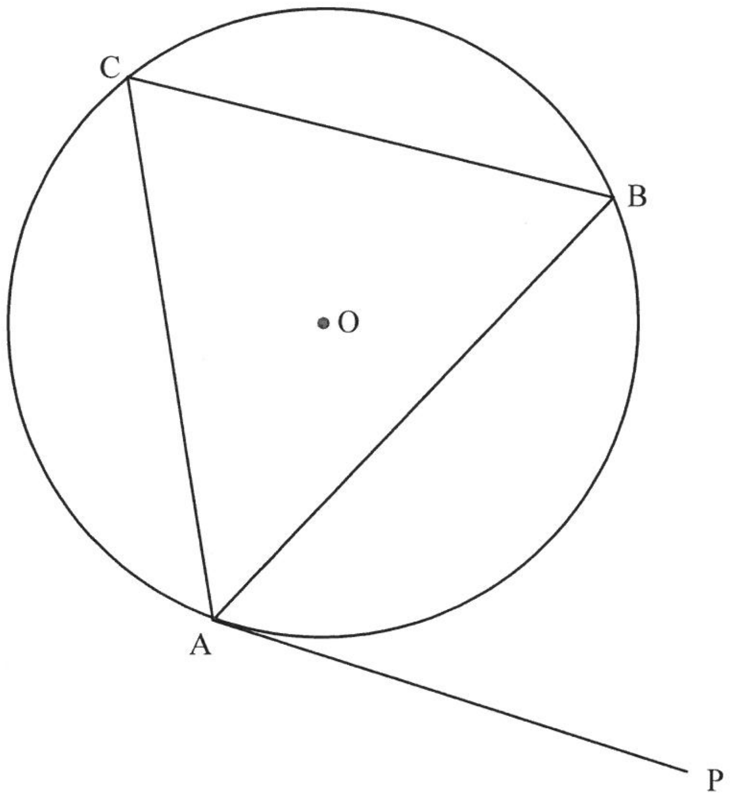
10.2.1 Prove that DE | | FC. (3)

10.2.2 If AB = 14 units, calculate the length of BF. (3)

[7]

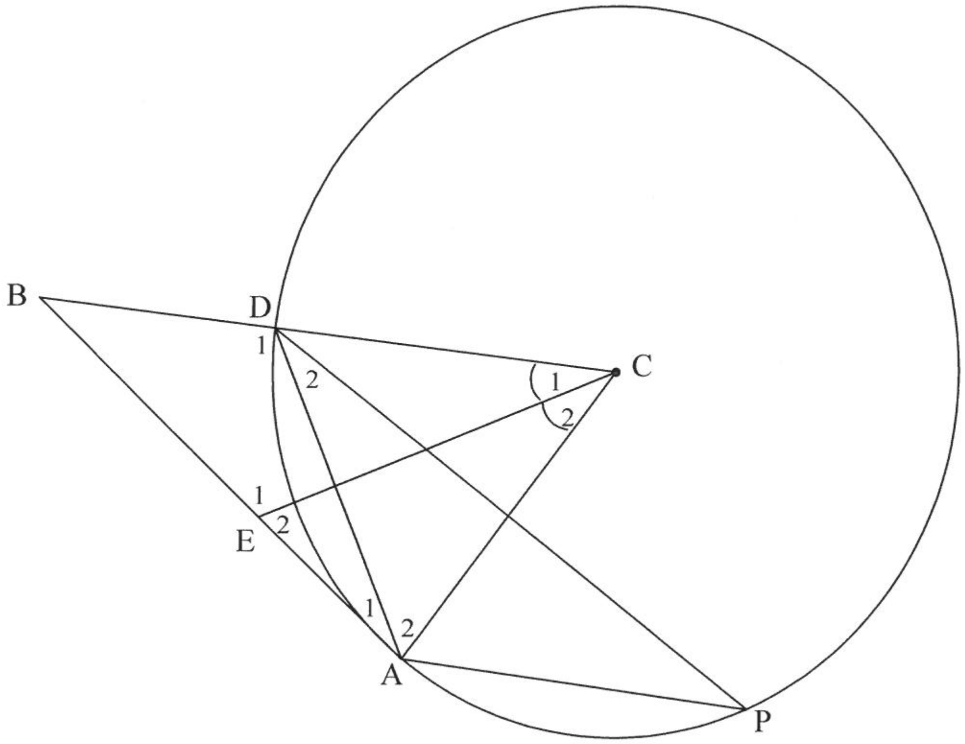
QUESTION 11

11.1 In the diagram O is the centre of the circle and PA is a tangent to the circle at A. B and C are points on the circumference of the circle.



Use the diagram to prove the theorem that states that $B\hat{A}P = A\hat{C}B$. (6)

11.2 In the diagram C is the centre of the circle DAP. BA is a tangent to the circle at A. CD is produced to meet the tangent to the circle at B. DP and DA are drawn. E is a point on BA such that EC bisects $D\hat{C}A$. Let $\hat{C}_1 = x$.



11.2.1 Prove that $\Delta BAD \mid\mid\mid \Delta BCE$. (7)

11.2.2 If it is also given that AB = 8 units and AC = 6 units, calculate:

(a) The length of BD (5)

(b) The length of BE (3)

(c) The size of x (3)
[24]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $A = P(1-ni)$ $A = P(1-i)^n$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $S_n = \frac{a(r^n - 1)}{r-1}$; $r \neq 1$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

area $\Delta ABC = \frac{1}{2} ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ P(A or B) = P(A) + P(B) – P(A and B)

$\hat{y} = a + bx$ $b = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$