basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATIONS/ NATIONAL SENIOR CERTIFICATE EXAMINATIONS

TECHNICAL MATHEMATICS P1

2019

MARKS: 150

TIME: 3 hours

This question paper consists of 10 pages, 2 answer sheets and a 2-page information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of NINE questions.
2. Answer ALL the questions.
3. Answer QUESTION 3.3, QUESTION 4.1.4, QUESTION 4.1.5 and QUESTION 4.3 on the ANSWER SHEETS provided. Write your centre number and examination number in the spaces provided on the ANSWER SHEETS and hand them in with your ANSWER BOOK.
4. Number the answers correctly according to the numbering system used in this question paper.
5. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.
6. Answers only will NOT necessarily be awarded full marks.
7. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
8. If necessary, round off answers to TWO decimal places, unless stated otherwise.
9. Diagrams are NOT necessarily drawn to scale.
10. An information sheet with formulae is included at the end of the question paper.
11. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $6x - 2x^2 = 0$ (3)

1.1.2 $x(2x+1) = 5$ (Round off to TWO decimal places.) (5)

1.1.3 $3x^2 \leq 12$ (4)

1.2 Solve for x and y if:

$$y + x = 4 \text{ en } x^2 - 3xy + y^2$$ (6)

1.3 The wheel-balancing machine is rotating a wheel at a rotational velocity, v, in m/s.

The formula for the rotational velocity of the wheel is given by:

$v = \frac{2\pi r}{t}$, where t is the time in seconds and r the radius of the wheel.

1.3.1 Express r as the subject of the formula. (2)

1.3.2 If $v = 91{,}116$ km/h and $t = 0{,}0000194$ hours:

(a) Write the time in scientific notation (2)

(b) Determine, in scientific notation, the value of r (in km) (2)

1.4 Convert 10111_2 to decimal number notation. (2)

[26]

QUESTION 2

2.1 Given: $E = \dfrac{1}{2p+7} + \sqrt{p+3}$

Determine for which value(s) of p will E be as follows:

2.1.1 Defined (1)

2.1.2 Real (2)

2.2 Given the equation: $kx^2 + 2 = 10x$

2.2.1 Determine the value of the discriminant in terms of k. (3)

2.2.2 Hence, determine the value of k, for which the roots of the equation will be equal. (2)

[8]

QUESTION 3

3.1 Simplify the following:

3.1.1 $3(2x)^0$ (1)

3.1.2 log (– 10) (1)

3.1.3 $\dfrac{5^{2n+1} + 4 \times 5^{2n}}{25^n}$ (3)

3.2 Solve for x: $\log_3(x+36) = \log_3 2x + \log 100$ (5)

3.3 Represent the complex number $z = 3 - 6i$ as an Argand diagram on the grid provided on the ANSWER SHEET. (3)

3.4 Solve for p and q if $p + qi = \dfrac{3-4i}{2+i}$ (5)

[18]

QUESTION 4

4.1 Given function f defined by $f(x) = -x^2 - 6x + 7$

4.1.1 Write down the y-intercept of f. (1)

4.1.2 Determine the x-intercepts of f. (3)

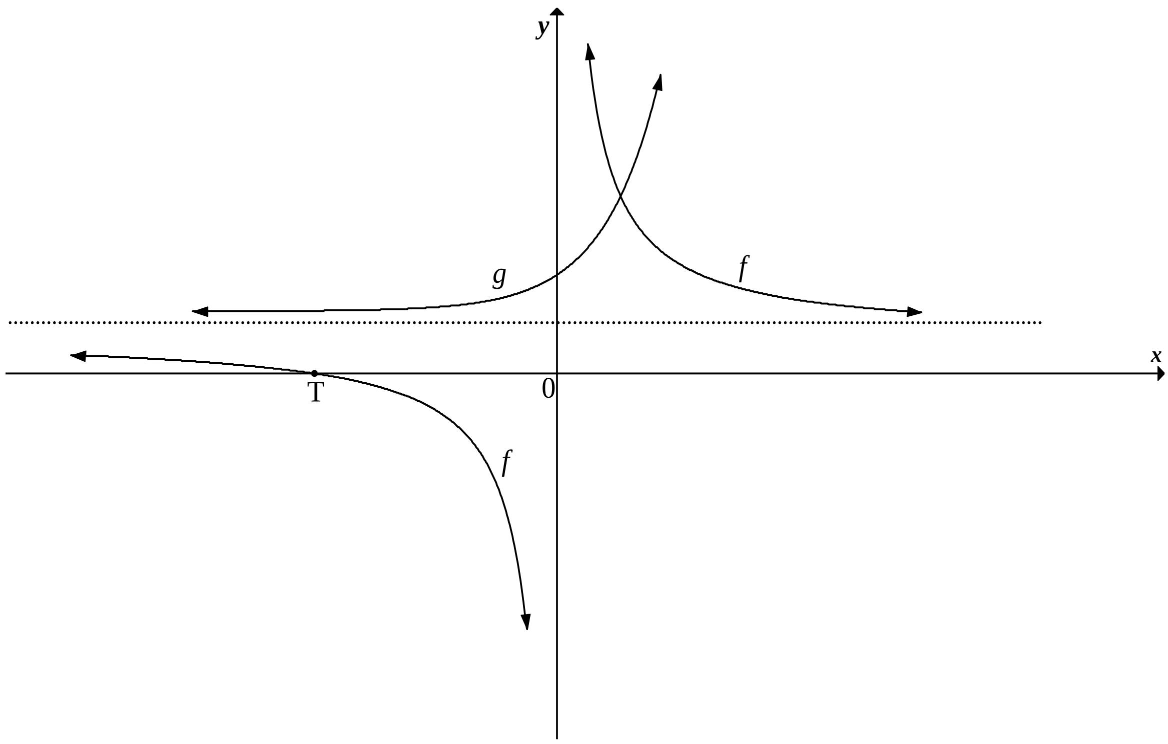
4.1.3 Determine the coordinates of the turning point of f. (3)

4.1.4 Hence, sketch the graph of f on the grid provided on the ANSWER SHEET. Clearly show the intercepts with the axes as well as the coordinates of the turning point of f. (3)

4.1.5 On the same set of axes, sketch the graph of h defined by $h(x) = 2x + 14$. Clearly show the intercepts with the axes. (2)

4.1.6 Determine the values of x for which $f(x) \geq h(x)$ (5)

4.2 The graph below represents functions f and g defined by $f(x) = \frac{4}{x} + 1$ and $g(x) = 3^x + q$ with f and g having a common asymptote.
T is the x-intercept of f.



4.2.1 Write down the numerical value of q. (1)

4.2.2 Determine the coordinates of T. (2)

4.2.3 Write down the domain of f. (1)

4.2.4 Write down the range of g. (1)

4.3 Given function p defined by $p(x) = -\sqrt{13 - x^2}$

Sketch the graph of p on the grid provided on the ANSWER SHEET. Show ALL the intercepts with the axes. (3)

[25]

QUESTION 5

5.1 The normal engine coolant temperature (ECT) of a certain car is 90 °C. Due to a technical problem, the engine of the car overheated which resulted in an ECT reading of 159 °C.

Determine the time (in minutes) it took the engine to cool down to the normal ECT if the temperature of the coolant cools down at a decreasing rate of 8% per minute. (4)

5.2 The graph below represents the compounded growth of an investment over a period of seven years. The interest rate for the first three years was 9,5% per annum. The dotted line represents the growth after an increase in the interest rate, while the solid line represents the growth when the interest rate remained unchanged.

5.2.1 Write down, from the graph, the initial amount invested. (1)

5.2.2 Determine (showing ALL calculations) the exact value of the investment at the end of the third year. (3)

5.2.3 After three years the interest rate of the investment changed to 12,5% per annum, compounded quarterly.

Determine how much more an investor will receive at the end of seven years, compared to the investment when the interest rate remained unchanged for the whole period. (6)

[14]

QUESTION 6

6.1 Determine $f'(x)$ using **first principles**, if $f(x) = 1 - x$ (5)

6.2 Determine:

6.2.1 $\frac{d}{dx}\left(2x^{-3} - 9x + 4\pi\right)$ (2)

6.2.2 $D_x\left[\frac{x^3 - 27}{x - 3}\right]$ (4)

6.2.3 $\frac{dy}{dx}$ if $xy = 7\sqrt{x}$ (3)

6.3 Given: $g(x) = 1 - x^3$

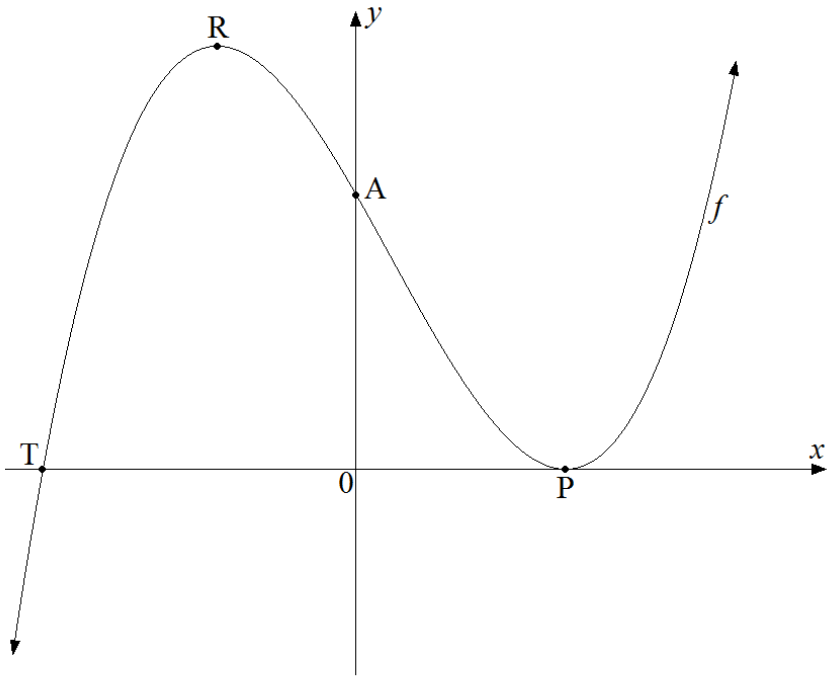
6.3.1 Determine the average gradient of g between the points where $x = -1$ and $x = 2$ (4)

6.3.2 Determine the equation of a tangent to the curve of g at the point where $y = 2$ (6)

[24]

QUESTION 7

The graph below represents the function defined by $f(x) = x^3 - x^2 - 8x + 12$
Points T, A and P are the intercepts of f on the axes.
Points R and P are the turning points of f.



7.1 Determine the length of OA. (2)

7.2 Show that $x - 2$ is a factor of $f(x)$. (2)

7.3 Factorise $f(x)$ completely. (3)

7.4 Hence, write down the coordinates of T and P. (2)

7.5 Determine the coordinates of R. (5)

7.6 Write down the values of x for which $f(x)$ is decreasing. (2)
[16]

QUESTION 8

The displacement (in metres) of a particle after t seconds is given by the formula:

$$s(t) = 7{,}5t^3 - 20t^2 + 27 \quad \text{where} \quad 0 \le t \le 5$$

Determine:

8.1 The initial displacement of the particle (1)

8.2 The instantaneous rate of change when $t = 3$ (3)

8.3 The time when the minimum displacement occurs (4)
[8]

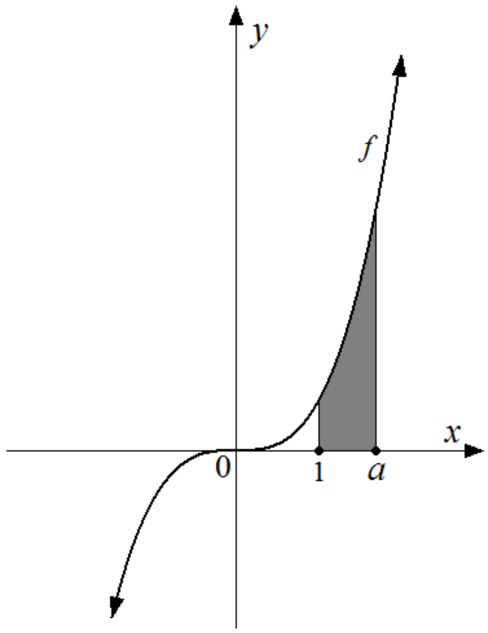
QUESTION 9

9.1 Determine the following integrals:

9.1.1 $\int (\pi x)\, dx$ (2)

9.1.2 $\int \left(x^{-1} - \sqrt{x} - 11\right) dx$ (4)

9.2 The diagram below shows the shaded area bounded by the curve defined by $g(x) = x^3$ and the x-axis between the points where $x = 1$ and $x = a$



Determine the value of a if the shaded area is 3,75 square units. (5)
[11]

TOTAL: 150

INFORMATION SHEET: TECHNICAL MATHEMATICS

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $x=-\frac{b}{2a}$ $y=\frac{4ac-b^2}{4a}$

$a^x=b \Leftrightarrow x=\log_a b$, $a>0$, $a\neq 1$ and $b>0$

$A=P(1+ni)$ $A=P(1-ni)$ $A=P(1-i)^n$ $A=P(1+i)^n$

$i_{eff}=\left(1+\frac{i}{m}\right)^m-1$

$f^{/}(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$\int x^n dx=\frac{x^{n+1}}{n+1}+C$, $n\neq -1$

$\int \frac{1}{x}dx=\ln x+C,$ $x>0$ $\int a^x dx=\frac{a^x}{\ln a}+C$, $a>0$

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $\mathrm{M}\left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$

$y=mx+c$ $y-y_1=m(x-x_1)$ $m=\frac{y_2-y_1}{x_2-x_1}$ $m=\tan\theta$

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

In ΔABC: $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$ $a^2=b^2+c^2-2bc\cdot\cos A$

area of $\Delta ABC=\frac{1}{2}ab\cdot\sin C$

$\sin^2\theta+\cos^2\theta=1$ $1+\tan^2\theta=\sec^2\theta$ $\cot^2\theta+1=\mathrm{cosec}^2\theta$

$\pi rad = 180°$

$\text{Angular velocity} = \omega = 2\pi n = 360°n$ where n = rotation frequency

$\text{Circumferencial velocity} = v = \pi Dn$ where D = diameter and n = rotation frequency

$s = r\theta$ where r = radius and θ = central angle in radians

$\text{Area of a sector} = \dfrac{rs}{2} = \dfrac{r^2\theta}{2}$ where r = radius, s = arc length and θ = central angle in radians

$4h^2 - 4dh + x^2 = 0$ where h = height of segment, d = diameter of circle and x = length of chord

$A_T = a(m_1 + m_2 + m_3 + ... + m_n)$ where a = equal parts, $m_1 = \dfrac{o_1 + o_2}{2}$ and n = number of ordinates

OR

$A_T = a\left(\dfrac{o_1 + o_n}{2} + o_2 + o_3 + o_4 + ... + o_{n-1}\right)$ where a = equal parts, o_i = i^{th} ordinate and n = number of ordinates

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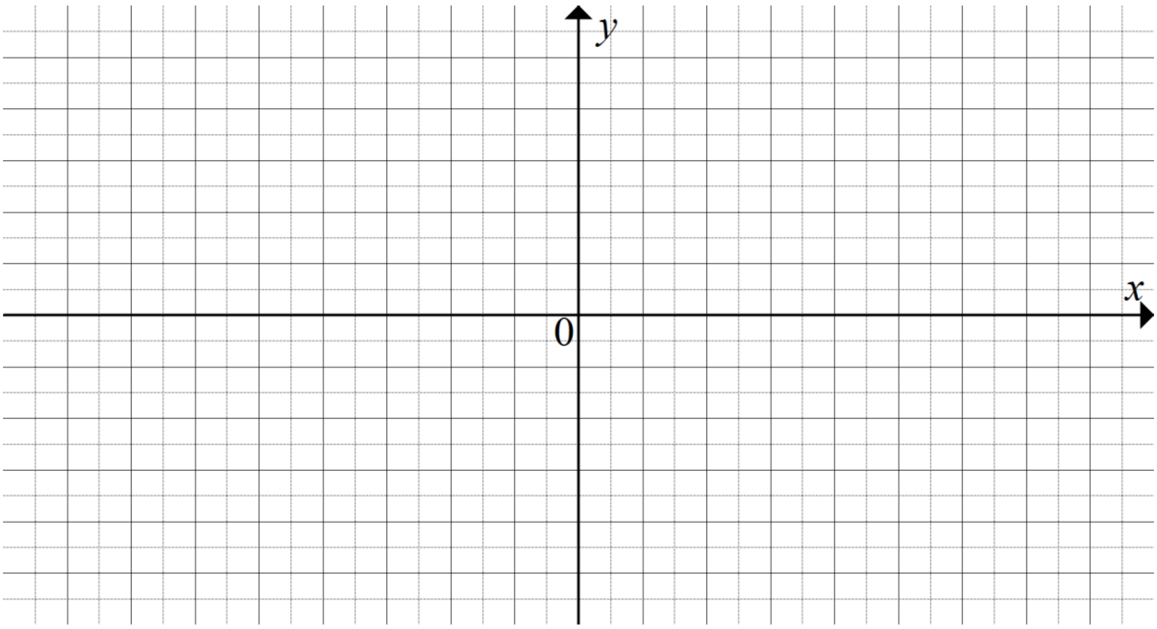
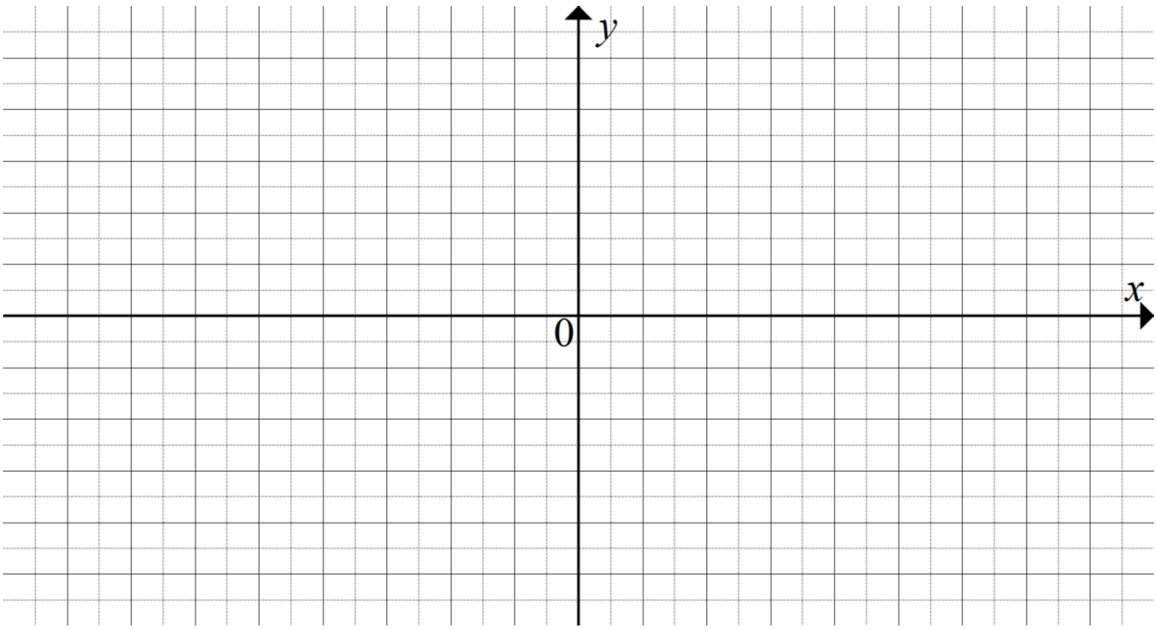
QUESTION 3.3



QUESTIONS 4.1.4 and 4.1.5



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QUESTION 4.3